Narrow Framing and Business Cycle Dynamics *

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Abstract

This paper studies how segmented information within firms affects the transmission of aggregate shocks. I develop a model in which each input manager observes their own input price but receives noisy signals about the other input manager's price and firm demand, which I refer to as "narrow framing". Although these frictions dampen firms' responses to input prices in partial equilibrium, they can amplify aggregate fluctuations by weakening offsetting general equilibrium feedback. In illustrative numerical exercises, capital managers' uncertainty about wages magnifies rental-rate movements and generates larger, more persistent capital responses to productivity shocks. Consumption responds less initially and more subsequently than under full information. Uncertainty about demand instead dampens the transmission of productivity shocks. These results show that the aggregate effects of information frictions depend on which decisions and feedback channels they affect.

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1 Introduction

Standard macroeconomic theory posits that firms make optimal production choices that maximize profits. An assumption that is often taken for granted is that, within a firm, all production decisions are jointly chosen under the same, perfect[1] information set to maximize a common objective of the firm, such as profits or dividends to shareholders. Under this framework, it is as if all the departments in the firm can perfectly coordinate the decisions that they are responsible for: because information can be perfectly shared within the firm, each department knows perfectly the information on which the other departments base their decisions. Hence, each department knows perfectly how its decision would affect others, and how the decisions of other departments would affect its own.

Although the assumption of perfect coordination greatly simplifies the analysis and provides a rational benchmark for firms' behavior, I argue that there are reasons to deviate from it. It is, however, inconsistent with findings in corporate finance and organizational economics. Since the firm's decision problems are often complicated, it is common for firms to delegate decision rights to different departments within the firm, where each department takes charge of a subset of decisions. Such a practice can lead to severe information problems within the firm. On the one hand, delegation may change departments' incentives to acquire information. Because of the delegation, departments tend to acquire more precise information about the decisions they are responsible for. The endogenous information acquisition of decision departments naturally leads to dispersed private information across the departments of the firm (Aghion and Tirole, 1997). On the other hand, delegation can create conflicts of interest among departments, and departments may have the incentive to strategically manipulate the information they transmit within the firm (Crawford and Sobel, 1982; Dessein, 2002). Other factors, such as geographical and cultural barriers within a multinational firm (Mian, 2006; Giroud, 2013) and the strength of relationships between managers and departments (Qian, Strahan, and Yang, 2015), may also contribute to inefficient information flows within the firm. The takeaway from this literature is that information asymmetry and communication frictions are pervasive within the firm, even with advances in organizational design (Malenko, 2022).

Meanwhile, there is a large macroeconomic literature on misoptimization and misallocation of resources by firms. The idea that firm managers engage in misoptimization due to cognitive and information constraints dates back to Simon (1955), and has been formal-

[1] Here, "perfect" means that the firm knows exactly the decision states, and "imperfect" means the firm has only a noisy representation of decision states.

ized more recently in the rational inattention literature (Maćkowiak and Wiederholt, 2009; Matějka, 2016). The novel textual evidence documented in Flynn and Sastry (2020) shows that there is a persistent, countercyclical misoptimization cycle among US public firms over the past four decades. At the aggregate level, the large literature on resource misallocation documents empirically that the firm-level marginal products of inputs are highly dispersed (Cooper and Haltiwanger, 2006; Foster, Haltiwanger, and Syverson, 2008), which can generate sizable misallocation and distortions even in the most advanced economies (Hsieh and Klenow, 2009). Although a number of existing works have studied the causes and consequences of misallocation and misoptimization in recent years (Hopenhayn, 2014; Midrigan and Xu, 2014; David, Hopenhayn, and Venkateswaran, 2016; David and Venkateswaran, 2019), the mechanisms that drive misallocation, the cyclical properties of misallocation, and the relationship between firm-level misoptimization and aggregate-level misallocation remain poorly understood (Restuccia and Rogerson, 2017).

In this paper, I propose a new mechanism, namely *narrow framing*, to model the information and coordination frictions within firms, and use it as a potential source of firm-level misoptimization and misallocation of resources across firms. Narrow framing is the tendency of people to truncate a complicated decision problem into several sub-problems and solve them separately without considering their full ramifications and effects on other decisions. The presence of narrow framing has been extensively documented in individual choice data. Experimental studies (Tversky and Kahneman, 1981; Rabin and Weizsäcker, 2009) show that, when faced with a pair of independent binary-choice problems, people tend to make decisions for each problem separately, even though such choices are less preferred when considered jointly. Narrow framing has been widely applied in many economic settings as a way of explaining non-optimizing behaviors, especially in consumption and finance research (see, for example, Thaler (1985); Barberis and Huang (2001)), but less so in the study of firms.

I follow the approach of Lian (2021) to operationalize narrow framing in the firm's decision problem. In my model, the firm delegates the input choices separately: capital decisions are delegated to a capital manager, while a labor manager is responsible for hiring decisions. The two managers jointly maximize the firm's profit. Because of common interest and strategic complementarity, both managers' marginal decisions depend on the decision of the other manager. The key friction is that the capital and labor managers in each firm operate on *different, non-nested and imperfect* information sets: I assume that the capital manager can perfectly observe the rental rate of capital, but receives a noisy, private signal about the wage rate, which is not directly relevant to the decision they are responsible for. Symmetrically, I assume that the labor manager can perfectly observe the wage rate, but only receives a noisy,

private signal about the rental rate of capital. With this formulation, the firm's decision problem becomes a static game of incomplete information between the capital manager and the labor manager, and the Bayesian Nash equilibrium of this Bayesian game characterizes the choice of capital by the capital manager and the choice of labor by the labor manager. As I will elaborate later, asymmetric information within the firm impedes both managers from reasoning through the implications of a shock for the other manager's decision. As a result, each manager's response to the other manager's decision is attenuated.

I separate my analysis into two parts. In the partial equilibrium analysis, I study how narrow framing affects the responsiveness of firms' input choices to input-price and demand shocks. I find that the elasticity of input choices to an input-price shock is dampened, whether the shock is own-price (i.e., a shock to the price that the manager knows perfectly) or cross-price (i.e., a shock to the price that the manager infers from a noisy signal). When hit by a cross-price shock, due to the noisy private signal about the other manager's price, the manager cannot distinguish between a cross-price shock and noise, and the signal extraction leads them to respond sluggishly to such shocks. When hit by an own-price shock, the attenuated response of the manager comes primarily from the fact that they know that the other manager would not fully respond to their own-price shock (for the other manager, it is a cross-price shock), and they internalize this in their decision making. I also study the responsiveness of the implied output and pricing decisions of the narrow-framing firm. I find that input-price shocks and demand shocks generate asymmetric responses in the firm's pricing decision: the former is dampened (under certain conditions) and the latter is amplified by narrow framing.

The second part of the partial equilibrium analysis shows that narrow framing can be regarded as a potential source of misoptimization at the firm level and misallocation of resources across firms. I find that under narrow framing, the firm-level misoptimization, defined as the wedge between a narrow-framing firm's input decision and a rational firm's input decision, can be partially predicted by the realized input prices and demand factors. In that sense, the misoptimization generated by narrow framing is state-dependent, which comes directly from the dampened responses of input choices to shocks. The idiosyncratic noise of capital and labor managers, on the other hand, is crucial for the dispersion of marginal product of capital and labor across the firms. It is responsible for both firm-specific capital-labor wedges and aggregate total factor productivity loss in the model.

Then, I analyze the implications of narrow-framing firms in general equilibrium. I embed narrow-framing firms into an otherwise standard real business cycle (RBC) model and study

how narrow framing would change the impulse response functions (IRFs) of endogenous variables to productivity and investment shocks. I find that narrow framing can significantly change the impact and propagation of these shocks in the RBC model. In the case of a productivity shock, I find that the noisy signals received by narrow-framing firms play different roles: the capital manager's noisy signal on wage and the labor manager's noisy signal on rental rate amplify the business cycle generated by the productivity shock compared to the rational benchmark, and the noisy signal on demand factors dampens the business cycle generated by the productivity shock. The explanation for this is that the former two frictions affect business cycle by dampening the feedback response of inputs to the movements of price of the other input, which often works in the opposite direction to and is dominated by the direct effect of a productivity shock, while the latter friction affects the business cycle primarily by attenuating the direct response of inputs to the shock.

Of the two amplifying signals, I find that the noisy wage signal changes the IRFs much more. This is mainly because the capital supply is predetermined, which generates a large, amplified movement in interest rate and hence in the consumption and capital decisions of the household in the case of noisy wage signal, while in the case of noisy rental rate signal, the effect of narrow framing is largely undone by the large movements in employment, which limits its ability to influence household behavior. In the case of an investment shock, I find that the noisy wage signal is the only friction that can generate sizable effects on endogenous variables in general equilibrium.

Related Literature First, this paper belongs to the literature that introduces information frictions into households' and firms' decision-making processes (Woodford, 2003; Maćkowiak and Wiederholt, 2009; Matějka, 2016; Gabaix, 2014; Flynn and Sastry, 2020). These papers typically assume that firms make decisions subject to a cognitive or attentional cost. Hence, in these papers, the decisions within the same firm are made under the same imperfect information. My paper is different from theirs in that in my model, these decisions are made under different, non-nested, imperfect information.

In terms of the modeling technique, my paper belongs to the literature that introduces incomplete information into macroeconomic models. The idea of modeling coordination and communication frictions by imperfect common knowledge is inherited directly from the influential work of Angeletos and coauthors (Angeletos and Pavan, 2007; Angeletos and La'O, 2013; Angeletos and Lian, 2016, 2018; Angeletos and Huo, 2021) and Lian (2021). The setup of a Bayesian game between multiple departments of the firm is similar to the game-theoretic

setup between multiple selves in behavioral economics (Piccione and Rubinstein, 1997). The idea of narrow framing is closely related to the mental budgeting and narrow bracketing literature in behavioral economics (Thaler, 1985; Heath and Soll, 1996; Thaler, 1999; Rabin and Weizsäcker, 2009; Hastings and Shapiro, 2013, 2018): as Lian (2021) points out, narrow framing provides a "smooth" model of mental accounting.

Lastly, this paper belongs to the misallocation literature that explores the relationship between misallocation and information frictions within and across firms. To the best of my knowledge, the closest paper in this literature is David, Hopenhayn, and Venkateswaran (2016), which allows the firms to choose labor and capital without having perfect information about the demand factor and finds that information friction is responsible for a considerable amount of misallocation and aggregate TFP loss. They also study a case where one of the input choices is chosen with perfect information while the choice of the other input is subject to uncertainty about demand factors. My model can be regarded as a generalization of theirs, because the model studied in their paper can be nested in mine by letting the capital and labor managers receive perfect signals about input prices and only noisy demand signals.

Layout The layout of this paper is as follows. Section 2 presents the firm's decision problem under narrow framing in detail, and discusses its main partial equilibrium implications for (1) the responsiveness of input choices to input-price and demand shocks, (2) misoptimization at the firm level and (3) misallocation of resources across firms. Section 3 studies an otherwise standard real business cycle model with narrow-framing firms. Section 4 concludes, and the Appendix contains the proofs.

2 Narrow Framing in Partial Equilibrium

In this section, I analyze the partial equilibrium response of a narrow-framing monopolistically competitive firm to a shock in input prices and demand.

2.1 Model

As is standard in the literature, the firm chooses price P, output Y, capital input K and labor input N to maximize profits

$$PY - RK - WN \tag{1}$$

subject to the production function

$$Y = F(K, N) \tag{2}$$

and the demand curve

$$P = D^{1/\epsilon} Y^{-1/\epsilon} \tag{3}$$

where R is the rental rate of capital, W is the nominal wage rate of labor, D is a demand shifter that is exogenous to the firm, and $\epsilon > 1$ denotes the elasticity of substitution of the firm's product in a CES aggregator (or equivalently, the demand elasticity of the good). I assume that the production function F is (1) strictly increasing in K and N, (2) supermodular (K and N are complements), and (3) strictly concave in K and N, i.e.,

$$\begin{aligned} F_k > 0, F_n > 0 \qquad & \text{(increasing)} \\ F_{kn} = F_{nk} > 0 \quad & \text{(complementarity)} \\ F_{kk} < 0, F_{nn} < 0, F_{kk}F_{nn} - F_{nk}F_{kn} > 0 \qquad & \text{(concave)} \end{aligned} \tag{4}$$

In the standard, rational benchmark, the firm chooses P, Y, K, N jointly, taking the input prices R, W and the demand shifter D as given. In other words, $\{R, W, D\}$ is common knowledge within the firm when each decision is made.

Information Now I introduce information and coordination frictions into the standard model. Assume that there is a manager who is in charge of the choice of capital K (I will call this manager "capital manager" hereafter), and there is a manager who is in charge of the choice of labor N (I will call this manager "labor manager"). After both managers choose K and N, the manufacturing department combines the chosen K and N to produce Y units of goods available for sale, according to the production function (2). After receiving the Y units of goods for sale, the retailing department sets the price P according to the demand function (3). As a result, given the capital manager's choice of capital K and the

labor manager's choice of labor N, the firm's profit is given by

$$D^{1/\epsilon}F(K,N)^{1-1/\epsilon} - RK - WN \tag{5}$$

The capital manager and labor manager choose K and N respectively to maximize (5), with the rental rate of capital R and wage rate W being common knowledge.

I assume that when the capital manager chooses capital, they observe the rental rate R perfectly but receive only a private, noisy signal about the wage rate W, denoted $\tilde{W}$. For tractability, assume that W has a log-normal prior distribution

$$\log W \sim N(\log \bar{W}, \sigma_w^2) \tag{6}$$

where $\bar{W}$ denotes the average (or steady-state) wage rate, and that the signal $\tilde{W}$ takes the form

$$\log \tilde{W} = \log W + \epsilon_w \tag{7}$$

where $\epsilon_w \sim N(0, \sigma_{\epsilon,w}^2)$ is Gaussian noise independent of W.

Similarly, I assume that when the labor manager chooses labor, they observe the wage rate W perfectly but receive only a private, noisy signal about the rental rate of capital R, denoted $\bar{R}$. The prior distribution of R and the signal $\tilde{R}$ take the same form as before:

$$\log R \sim N(\log \bar{R}, \sigma_r^2) \tag{8}$$

$$\log \tilde{R} = \log R + \epsilon_r \tag{9}$$

where $\bar{R}$ denotes the average rental rate and $\epsilon_r \sim N(0, \sigma_{\epsilon,r}^2)$ is Gaussian noise independent of R.

I assume that both managers receive the same noisy signal $\tilde{D}$ about the aggregate demand D, with prior and noise in the form of

$$\log D \sim N(\log \bar{D}, \sigma_d^2) \tag{10}$$

$$\log \tilde{D} = \log D + \epsilon_d \tag{11}$$

where $\bar{D}$ denotes the average aggregate demand and $\epsilon_d \sim N(0, \sigma_{\epsilon,d}^2)$ is Gaussian noise independent of D.

The assumption that each manager receives a perfect signal about their own price and

receives a noisy signal about the other manager's price follows directly from Lian (2021). The assumption is also consistent with the observation by Aghion and Tirole (1997): managers acquire more precise information about the decision states that relate to the decisions they are responsible for. As we will see soon, the noisy signal about the other manager's price impedes the managers from reasoning through the ramifications of the other manager's decision on their own decisions. For instance, consider the effect of an increase in wage rate on capital choice. Since wage rate does not enter the capital manager's first-order condition directly (as opposed to the rental rate of capital), it affects capital choice only through the capital manager's guess of how the labor manager would respond to the change in wage rate. However, since their information about the wage rate is noisy, they cannot tell whether the increase in the wage signal results from a wage increase or from noise, so their guess about the labor manager's response to the wage shock is imperfect. I discuss the implications of these frictions in detail in the following subsections.

In this paper, I do not take a stand on where these noisy signals come from. There are several possible origins of noisy information in the literature. The macroeconomic literature on firm dynamics typically uses rational inattention (e.g., Sims (2003), Maćkowiak and Wiederholt (2009), and more recently, Flynn and Sastry (2020)) to endogenously generate noisy, dispersed information among agents. The corporate finance literature mainly adopts the approach of Crawford and Sobel (1982) to model strategic information transmission within organizations. Whether inattentiveness or strategic information transmission should be responsible for the noisy, dispersed information within the firm requires further research.

In sum, the signal that the capital manager receives is summarized by the vector $s^k \equiv (R, \tilde{W}, \tilde{D})$, and the signal that the labor manager receives is $s^n \equiv (\tilde{R}, W, \tilde{D})$. I denote the σ-algebra generated by s^k and s^n as information set $\mathcal{I}^k$ and $\mathcal{I}^n$, respectively. I assume that the prior distributions (6), (9), (10), as well as the distribution of signals (7), (9) and (11), are commonly known to both managers. In other words, each manager i knows that they receive a noisy signal about the other manager $-i$'s prices, and manager $-i$ knows that manager i receives a noisy signal about manager $-i$'s prices, and manager i knows that manager $-i$ knows that, and so on. An advantage of this formulation is that once each manager receives their own private signals, it is easy to characterize their beliefs about the signals that the other manager receives, conditional on the realization of their own signals. It also ensures that the Bayesian game and the Bayesian Nash equilibrium characterized below are well defined.

Game and Equilibrium With asymmetric information between the capital manager and labor manager, once the capital manager and the labor manager receive their own private signals, they simultaneously choose the decision that they are responsible for, conditional on the realization of their own signals, to maximize the firm's profit (5). The restriction is that each manager's decision can only be measurable with respect to their own signals, not the signals that the other manager receives.

The decision process that I described above can be naturally formulated as a game of incomplete information between the capital manager and the labor manager, with the information sets defined above and the payoff function given by (5). The equilibrium behaviors of the capital and labor manager can therefore be characterized by the Bayesian Nash equilibrium of the game. Formally, let $K(\cdot) : \mathcal{I}^k \to \mathbb{R}_+$ and $N(\cdot) : \mathcal{I}^n \to \mathbb{R}_+$ be the decision rule of the capital manager and the labor manager respectively. The decision rules that constitute a Bayesian Nash equilibrium can be characterized as follows:

Definition 1. Capital manager's decision rule $K(\cdot) : \mathcal{I}^k \to \mathbb{R}_+$ and labor manager's decision rule $N(\cdot) : \mathcal{I}^n \to \mathbb{R}_+$ constitute a Bayesian Nash equilibrium if

1. Given labor manager's decision rule $N(\cdot)$, capital manager's decision rule $K(s^k)$ satisfies

$$K(s^k) \in \arg\max_K E\left[D^{1/\epsilon}F(K, N(s^n))^{1-1/\epsilon} - RK - WN(s^n)\middle| s^k\right] \tag{12}$$

   for all s^k, and

2. Given capital manager's decision rule $K(\cdot)$, labor manager's decision rule $N(s^n)$ satisfies

$$N(s^n) \in \arg\max_N E\left[D^{1/\epsilon}F(K(s^k), N)^{1-1/\epsilon} - RK(s^k) - WN\middle| s^n\right] \tag{13}$$

   for all s^n.

Essentially, asymmetric information between the capital manager and labor manager impedes the managers from reasoning through the decision of the other manager. Because the managers receive only noisy signals about the other manager's input price, they do not know exactly the input price that the other manager is facing, and hence do not know exactly what the other manager would play. Hence, upon the realization of their own signals, the managers can only form posterior beliefs about what the other manager would play conditional on that manager's private signals. Bayesian Nash equilibrium requires that each manager chooses the best option that maximizes the "perceived" profit of the firm using the posterior.

The Bayesian Nash equilibrium in Definition 1 nests the behavior of a standard, "rational" firm. For a rational firm, information is frictionlessly shared and communicated within the firm, so that rental rate of capital R, wage rate W and demand factor D are common knowledge for all entities within the firm. As a result, the capital and labor managers operate on the same information set that contains the true value of R, W and D. Using the notation in Definition 1, we have $s^m = s^p = s \equiv \{R, W, D\}$, and the optimal decision rule $\{K^{rational}(\cdot), N^{rational}(\cdot)\}$ maximizes the profit function (5) for each realization of s. Another way to interpret the rational benchmark is to regard it as the limiting case where the idiosyncratic signals that the managers receive are perfectly precise, i.e., $\sigma^2_{\epsilon,i} \to 0$ for $i = r, w, d$. Intuitively, as the signals get more informative towards the limiting case (and this is commonly known), the managers would behave as if they are both operating with perfect knowledge of input prices and demand factors.

Although the Bayesian game and Bayesian Nash equilibrium above are less restrictive than the rational, complete information benchmark in terms of information structure, I should point out that attaining the Bayesian Nash equilibrium in Definition 1 still requires a considerable amount of rationality from the capital and labor manager. Both managers are presumed to have an infinite regress of higher-order beliefs about their "errors": manager i makes a decision, knowing that the other manager $-i$ makes a decision without knowing perfectly i's decision, and that manager $-i$ knows that manager i knows that, and so on. Based on their private information, they best respond to beliefs of their opponent's play, and in equilibrium, their beliefs are correct in the sense that they are consistent with both managers' optimal decision rules under such beliefs. Whether this equilibrium assumption holds in reality remains an open question.

Decision Rules under Narrow Framing Now I characterize the optimal decision rules $\{K(\cdot), N(\cdot)\}$ for a narrow-framing firm. Taking first-order conditions of (12) and (13) yields

$$E\left[D^{1/\epsilon}F(K(s^k), N(s^n))^{-1/\epsilon}F_k(K(s^k), N(s^n))\middle| s^k\right] = \frac{\epsilon}{\epsilon - 1}R \tag{14}$$

$$E\left[D^{1/\epsilon}F(K(s^k), N(s^n))^{-1/\epsilon}F_n(K(s^k), N(s^n))\middle| s^n\right] = \frac{\epsilon}{\epsilon - 1}W \tag{15}$$

Let $\bar{K}, \bar{N}$ denote the rational firm's choice of capital and labor when the input prices and aggregate demand are at their average value, $\bar{R}, \bar{W}, \bar{D}$. Log-linearizing the first-order condi-

tions (14) and (15) gives

$$E\left[\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)k+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)n+\frac{1}{\epsilon}d\middle|s^k\right] = r \tag{16}$$

$$E\left[\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)k+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)n+\frac{1}{\epsilon}d\middle|s^n\right] = w \tag{17}$$

where the lower-case letter $x \equiv \log X - \log \bar{X}$ denotes the log-deviation of X from its average value $\bar{X}$ for $X = K, N, D, R, W$, and $\gamma_{kk}, \gamma_{kn}, \gamma_{nk}, \gamma_{nn}, \gamma_k, \gamma_n$ are constants that can be written as functions of average variables:

$$\gamma_{kk} = \frac{F_{kk}(\bar{K},\bar{N})\bar{K}}{F_k(\bar{K},\bar{N})}, \gamma_{kn} = \frac{F_{kn}(\bar{K},\bar{N})\bar{N}}{F_k(\bar{K},\bar{N})}, \gamma_k = \frac{F_k(\bar{K},\bar{N})\bar{K}}{F(\bar{K},\bar{N})},$$

$$\gamma_{nk} = \frac{F_{nk}(\bar{K},\bar{N})\bar{K}}{F_n(\bar{K},\bar{N})}, \gamma_{nn} = \frac{F_{nn}(\bar{K},\bar{N})\bar{N}}{F_n(\bar{K},\bar{N})}, \gamma_n = \frac{F_n(\bar{K},\bar{N})\bar{N}}{F(\bar{K},\bar{N})}.$$

With our assumptions on the production function in (4), it is immediate that $\gamma_{kk} < 0, \gamma_{kn} > 0, \gamma_{nk} > 0, \gamma_{nn} < 0, \gamma_k > 0, \gamma_n < 0$, and $\gamma_{kk}\gamma_{nn} - \gamma_{kn}\gamma_{nk} > 0$. For the details of log-linearization, please consult Appendix A.1.

Since the logged signals are Gaussian, we can easily characterize the partial equilibrium decision rule for a narrow-framing firm according to (16) and (17). The optimal log-linearized decision rule takes the form

$$k = \alpha_r(\lambda_r)r + \alpha_w(\lambda_w)\tilde{w} + \alpha_d(\lambda_d)\tilde{d} \tag{18}$$

$$n = \beta_r(\lambda_r)\tilde{r} + \beta_w(\lambda_w)w + \beta_d(\lambda_d)\tilde{d} \tag{19}$$

where $\tilde{r}, \tilde{w}, \tilde{d}$ are the log deviations of signals $\tilde{R}, \tilde{W}, \tilde{D}$ from their average values $\bar{R}, \bar{W}, \bar{D}$ respectively, and

$$\lambda_r \equiv \frac{\sigma_{\epsilon,r}^{-2}}{\sigma_{\epsilon,r}^{-2}+\sigma_r^{-2}}, \lambda_w \equiv \frac{\sigma_{\epsilon,w}^{-2}}{\sigma_{\epsilon,w}^{-2}+\sigma_w^{-2}}, \lambda_d \equiv \frac{\sigma_{\epsilon,d}^{-2}}{\sigma_{\epsilon,d}^{-2}+\sigma_d^{-2}}$$

are the Kalman gains of the signals $\tilde{r}, \tilde{w}, \tilde{d}$ respectively. Details of the derivation are relegated to Appendix A.2.

In the optimal log-linearized decision rule (18) and (19), the coefficients $\alpha_i(\lambda_i), \beta_i(\lambda_i)$ are elasticities of input choices with respect to changes in the private signals that the managers receive, and they can be expressed as a function of the Kalman gains $\lambda_r, \lambda_w, \lambda_d$. These

Kalman gains can be interpreted as a measure of informativeness of the signals, as well as the degree of information friction within the firm: the larger they are, the more weight managers place on the signals they receive, and the less frictional information flow and communication within the firm are. We are interested in how information frictions would affect the responsiveness of input choices to a change in input prices and aggregate demand, and in particular, how the responsiveness of a narrow-framing firm compares to that of a "rational" firm that is free from information frictions. As I previously mentioned, such a comparison is easy because the case of rational firms can be nested in the current model, where both managers receive perfect signals about r, w, d. As the signal gets more precise, $\sigma^2_{\epsilon,i} \to 0$, and hence the case of rational firms maps to the limiting case $\lambda_i \to 1$ in our model. The other limiting case, where the managers receive extremely noisy signals, so that $\sigma^2_{\epsilon,i} \to \infty$ and $\lambda_i \to 0$, maps to the case where the managers do not pay attention to the noisy signals and solely rely on the prior to make their decisions.

Note that even though the managers receive perfect information about their own prices, their own-price elasticities, α_r and β_w, depend on the informativeness of signals that the other manager receives. This results exactly from the equilibrium assumption: the managers know that the other manager receives a noisy signal about their own prices, they know that the other manager has trouble thinking through a shock in their own prices, and they internalize that in their decisions.

Conditional on the true input prices and aggregate demand r, w, d, the average capital and labor demand can be easily obtained from the signal-specific decision rules (18) and (19):

$$E[k|r, w, d] = \alpha_r(\lambda_r)r + \alpha_w(\lambda_w)w + \alpha_d(\lambda_d)d \tag{20}$$

$$E[n|r, w, d] = \beta_r(\lambda_r)r + \beta_w(\lambda_w)w + \beta_d(\lambda_d)d \tag{21}$$

Hence, the coefficients $\alpha_i(\lambda_i), \beta_i(\lambda_i)$ also denote the average or aggregate elasticities of input choices to the true prices and demand shocks r, w, d, and (20) and (21) can be regarded as the demand functions for capital and labor, respectively.

2.2 Main Predictions

2.2.1 Propagation of Input-price Shocks and Demand Shocks

Now I present the main predictions of the narrow framing model on the responsiveness of input choices to shocks in input prices and aggregate demand. All the proofs in the main text can be found in Appendix A.3.

The first result compares how a narrow-framing firm and a rational firm adjust capital in response to a change in the wage rate, and labor in response to a change in the rental rate:

Proposition 1 (Cross-price Elasticities)**.** *A narrow-framing firm's response of capital to a change in the wage rate is dampened, and a narrow-framing firm's response of labor to a change in the rental rate is dampened. Formally,*

1. *If* $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k \geq 0$*, then*

$$\frac{\partial n^{rational}}{\partial r} \leq \frac{\partial n^{narrow}}{\partial r} < 0 \tag{22}$$

otherwise,

$$0 < \frac{\partial n^{narrow}}{\partial r} < \frac{\partial n^{rational}}{\partial r} \tag{23}$$

2. *If* $\gamma_{kn} - \frac{1}{\epsilon}\gamma_n \geq 0$*, then*

$$\frac{\partial k^{rational}}{\partial w} \leq \frac{\partial k^{narrow}}{\partial w} < 0 \tag{24}$$

otherwise,

$$0 < \frac{\partial k^{narrow}}{\partial w} < \frac{\partial k^{rational}}{\partial w} \tag{25}$$

The reason why the cross-price elasticities of input choices are dampened can be seen from the log-linearized first-order conditions (16) and (17). Applying the implicit function theorem to (16) yields

$$\frac{\partial k}{\partial w} = -\frac{\gamma_{kn} - \frac{1}{\epsilon}\gamma_n}{\gamma_{kk} - \frac{1}{\epsilon}\gamma_k}\frac{\partial E\left[n|s^k\right]}{\partial w} \tag{26}$$

i.e., a wage change affects capital choice only via the capital manager's expectation of how the labor manager would change their hiring decision. However, since the capital manager receives only a noisy signal about the wage, they cannot tell whether a change in their signal $\tilde{w}$ reflects a change in the wage or noise. They must solve a signal-extraction problem to infer the wage change from the signal they receive, and as a result, their reasoning about

how a wage change would affect the labor manager's hiring decision is attenuated:

$$\left|\frac{\partial E[n|s^k]}{\partial w}\right| = \left|\frac{\partial}{\partial w}E[\beta_r\tilde{r} + \beta_w w + \beta_d\tilde{d}|r, \tilde{w}, \tilde{d}]\right| = \lambda_w|\beta_w| \leq \left|\frac{\partial n}{\partial w}\right| \tag{27}$$

Therefore, the response of their capital choice to a change in the wage rate is dampened in magnitude, regardless of the sign of the elasticity.

The next result establishes the comparison of a narrow-framing firm's and a rational firm's responses of capital to a change in the rental rate, and those of labor to a change in the wage rate:

Proposition 2 (Own-price Elasticities)**.** *A narrow-framing firm's response of capital to a change in the rental rate is dampened, and a narrow-framing firm's response of labor to a change in the wage rate is dampened. Formally,*

$$\frac{\partial k^{rational}}{\partial r} \leq \frac{\partial k^{narrow}}{\partial r} < 0 \tag{28}$$

and

$$\frac{\partial n^{rational}}{\partial w} \leq \frac{\partial n^{narrow}}{\partial w} < 0 \tag{29}$$

To interpret this result, we again return to the log-linearized first-order conditions (16) and (17). Applying the implicit function theorem to (16) and (17) yields

$$\frac{\partial k}{\partial r} = \underbrace{\frac{1}{\gamma_{kk} - \frac{1}{\epsilon}\gamma_k}}_{\text{direct effect}} + \underbrace{\frac{\gamma_{kn} - \frac{1}{\epsilon}\gamma_n}{-\gamma_{kk} + \frac{1}{\epsilon}\gamma_k}\frac{\partial E\left[n|s^k\right]}{\partial r}}_{\text{indirect effect}} \tag{30}$$

Equation (30) decomposes the effect of a rental-rate change on capital into two parts. The first part is the direct response of capital to a rental-rate change: an increase in rental rate makes capital more expensive, hence the capital manager would lower the capital stock. As we can see, since we assumed in (4) that the production function is concave and increasing, the direct effect in (30) is always negative. Since I assumed that the capital manager receives a perfect signal about the rental rate, the direct effect is the same for the narrow-framing firm and the rational firm.

The second term in (30) is the indirect effect from the "higher-order" or strategic reasoning of the capital manager: it captures the capital manager's response to their guesses about how the labor manager would respond to a change in the rental rate. The direction of the

indirect effect is controlled by the sign of $\gamma_{kn} - \frac{1}{\epsilon}\gamma_n$ and $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k$. From Proposition 1, we know that the sign of capital manager's perceived rental rate elasticity of labor choice, $\frac{\partial E[n|s^k]}{\partial r}$, is negative if and only if $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k$ is positive. If the coefficient on the perceived elasticity, $\frac{\gamma_{kn} - \frac{1}{\epsilon}\gamma_n}{-\gamma_{kk} + \frac{1}{\epsilon}\gamma_k}$, is also positive, so that $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) > 0$, then the indirect effect is negative and in the same direction as the direct effect. From Proposition 1, we know that, due to the information friction, the indirect effect under narrow framing will be attenuated. As a result, the overall effect of a rental rate change on capital choice is attenuated. If, on the other hand, the coefficient on the perceived elasticity is negative, so that $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) < 0$, then the indirect effect is positive and works in the opposite direction of the direct effect, and hence the overall effect will be amplified. However, this will never be the case, since our assumption on production function (4) implies that

$$\frac{\gamma_k}{\gamma_{nk}} = \frac{\gamma_n}{\gamma_{kn}} = \frac{F_k(\bar{K}, \bar{N})F_n(\bar{K}, \bar{N})}{F(\bar{K}, \bar{N})F_{kn}(\bar{K}, \bar{N})}$$

and therefore $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) \geq 0$ for all ϵ.

The argument for the case where $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k$ is negative and the perceived rental rate elasticity of labor choice $\frac{\partial E[n|s^k]}{\partial r}$ is positive is the same as above: when $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k < 0$, from Proposition 1, we know that $\frac{\partial E[n|s^k]}{\partial r} > 0$. And since $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) \geq 0$, the coefficient on the perceived rental rate elasticity of labor choice, $\frac{\gamma_{kn} - \frac{1}{\epsilon}\gamma_n}{-\gamma_{kk} + \frac{1}{\epsilon}\gamma_k}$, is negative. Therefore, in this case the indirect effect is again negative and in the same direction as the direct effect. The dampened indirect effect due to narrow framing therefore leads to attenuation of the overall effect. As a result, the own-price elasticities of input choices are always dampened under narrow framing.

As we can see, the analysis of own-price elasticity is more involved than that of cross-price elasticity. While we merely need to consider the impact of narrow framing on the strategic reasoning for the cross-price results, to get a full picture of own-price elasticity requires a comparison of both the direction and the strength of direct and indirect effects. In Proposition 2, the analysis is simplified since our assumption (4) on the production function guarantees that $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) \geq 0$, which implies that the indirect effect is always in the same direction as the direct effect. If, on the other hand, it is possible for the indirect effect to work in the opposite direction to the direct effect, then whether the overall effect is amplified or attenuated depends on which effect is dominating: if the direct effect is dominating, so that the overall effect is in the same direction as the direct effect, then narrow framing amplifies the overall effect. If the indirect effect is dominating, then narrow

framing will again dampen the overall effect.

Lastly, I establish the elasticities of capital and labor choice to a change in aggregate demand.

Proposition 3 (Demand Elasticities)**.** *A narrow-framing firm's response of capital and labor to a change in aggregate demand is dampened, i.e.,*

$$0 < \frac{\partial k^{narrow}}{\partial d} < \frac{\partial k^{rational}}{\partial d} \tag{31}$$

and

$$0 < \frac{\partial n^{narrow}}{\partial d} < \frac{\partial n^{rational}}{\partial d} \tag{32}$$

This result follows immediately from the fact that both managers receive the same noisy signal about aggregate demand. Since they cannot distinguish between a demand shock and noise, their responses to the demand shock are attenuated.

Implied Output and Prices Apart from the elasticity of input choices to input-price shocks and demand shocks, we may also be interested in how narrow framing would impact the firm's output and pricing decisions.

With the optimal decision rule for capital and labor as in (18) and (19), we can immediately compute the output of the narrow-framing firm. Let $y \equiv \log F(K, N) - \log F(\bar{K}, \bar{N})$ denote the log deviation of output from its rational (complete-information) benchmark. Output y is connected to input choices k and n by the log-linearized production function

$$y = \gamma_k k + \gamma_n n \tag{33}$$

and therefore we can easily obtain the elasticity of output (averaged over realizations of signals, conditional on the true r, w, d) to the input price shocks and demand shocks:

$$\frac{\partial E[y|r,w,d]}{\partial r} = \gamma_k \alpha_r(\lambda_r) + \gamma_n \beta_r(\lambda_r) \tag{34}$$

$$\frac{\partial E[y|r,w,d]}{\partial w} = \gamma_k \alpha_w(\lambda_w) + \gamma_n \beta_w(\lambda_w) \tag{35}$$

$$\frac{\partial E[y|r,w,d]}{\partial d} = \gamma_k \alpha_d(\lambda_d) + \gamma_n \beta_d(\lambda_d) \tag{36}$$

The following proposition compares a narrow-framing firm's and a rational firm's response of output to input price shocks and demand shocks.

Proposition 4 (Elasticity of Output)**.** *The response of output to input price shocks and demand shocks has the following pattern:*

1. *On the elasticity of output to a change in the rental rate of capital:*

   - *if* $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k > 0$*, then the response of output to a change in the rental rate of capital is attenuated:*

$$\frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial r} < \frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial r} < 0 \tag{37}$$

   - *otherwise, the response of output to a change in the rental rate of capital is amplified:*

$$\frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial r} < \frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial r} < 0 \tag{38}$$

2. *On the elasticity of output to a change in the wage rate:*

   - *if* $\gamma_{kn} - \frac{1}{\epsilon}\gamma_n > 0$*, then the response of output to a change in the wage rate is attenuated:*

$$\frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial w} < \frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial w} < 0 \tag{39}$$

   - *otherwise, the response of output to a change in the wage rate is amplified:*

$$\frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial w} < \frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial w} < 0 \tag{40}$$

3. *The response of output to a change in demand is attenuated:*

$$0 < \frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial d} < \frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial d} \tag{41}$$

As we can see, whether the output is more or less responsive to input price shocks depends on the relationship between demand elasticity ϵ and the production function F, and this can be easily interpreted by our intuition built in Propositions 1 and 2. Consider the case of an increase in the rental rate of capital. Equation (34) states that its overall effect on output is a weighted sum of its effect on capital and labor. From Proposition 2, a rate hike will always lead to a dampened, negative effect on capital for a narrow-framing firm. From Proposition 1, if $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k > 0$, a rate hike leads to a dampened, negative effect on labor for a narrow-framing firm. In this case, the rental rate of capital affects capital and labor in the same direction, and therefore the overall effect on the output will be dampened. If, on the

other hand, $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k < 0$, a rate hike leads to a dampened, positive effect on labor for a narrow-framing firm, according to Proposition 1. In this case, capital and labor are affected in opposite directions by the change in rental rate of capital, and therefore the overall effect on output will be amplified.

Lastly, I establish the results for prices. Log-linearizing the demand function (3) gives

$$p = \frac{1}{\epsilon}(d - y) \tag{42}$$

Therefore, the elasticity of price to the input price shocks and demand shocks can be written as

$$\frac{\partial E[p|r,w,d]}{\partial r} = -\frac{1}{\epsilon}\frac{\partial E[y|r,w,d]}{\partial r} \tag{43}$$

$$\frac{\partial E[p|r,w,d]}{\partial w} = -\frac{1}{\epsilon}\frac{\partial E[y|r,w,d]}{\partial w} \tag{44}$$

$$\frac{\partial E[p|r,w,d]}{\partial d} = \frac{1}{\epsilon}\left(1 - \frac{\partial E[y|r,w,d]}{\partial d}\right) \tag{45}$$

The following proposition is immediate:

Proposition 5 (Elasticity of Price)**.** *The response of price to input price shocks and demand shocks has the following pattern:*

1. *On the elasticity of price to a change in the rental rate of capital:*

   - *if $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k > 0$, then the response of price to a change in the rental rate of capital is attenuated:*

   $$\frac{\partial E\left[p^{rational}|r,w,d\right]}{\partial r} > \frac{\partial E\left[p^{narrow}|r,w,d\right]}{\partial r} > 0 \tag{46}$$

   - *otherwise, the response of price to a change in the rental rate of capital is amplified:*

   $$\frac{\partial E\left[p^{narrow}|r,w,d\right]}{\partial r} > \frac{\partial E\left[p^{rational}|r,w,d\right]}{\partial r} > 0 \tag{47}$$

2. *On the elasticity of price to a change in the wage rate:*

   - *if $\gamma_{kn} - \frac{1}{\epsilon}\gamma_n > 0$, then the response of price to a change in the wage rate is attenuated:*

   $$\frac{\partial E\left[p^{rational}|r,w,d\right]}{\partial w} > \frac{\partial E\left[p^{narrow}|r,w,d\right]}{\partial w} > 0 \tag{48}$$

- *otherwise, the response of price to a change in the wage rate is amplified:*

$$\frac{\partial E\left[p^{narrow}|r,w,d\right]}{\partial w} > \frac{\partial E\left[p^{rational}|r,w,d\right]}{\partial w} > 0 \tag{49}$$

3. *The response of price to a change in demand is amplified:*

$$\frac{\partial E\left[p^{narrow}|r,w,d\right]}{\partial d} > \frac{\partial E\left[p^{rational}|r,w,d\right]}{\partial d} \geq 0 \tag{50}$$

The results on the elasticity of price to input shocks come directly from Proposition 4 and Equations (43) and (44): since the response of price to input shocks is proportional to the response of output to input shocks, the conclusions in Proposition 4 can be directly applied to the elasticity of price. The last part states that the demand elasticity of price will always be amplified for a narrow-framing firm. From (45) we know that the effect of a demand shock on price can be decomposed into a direct effect and an indirect effect via output. An increase in aggregate demand will push up the price that the firm sets. However, the increase in aggregate demand will also increase outputs, which introduces a negative indirect effect on price. In this case, the direct effect and indirect effect move in opposite directions, and a dampened indirect effect introduced by narrow framing amplifies the response of the firm's price to demand shocks.[2]

2.2.2 Misoptimization and Misallocation

In this subsection I explore the implications of narrow framing for the misoptimization of individual firms and misallocation on the aggregate level. For illustrative purposes, I restrict my attention to the constant-returns-to-scale Cobb-Douglas production function

$$Y_{it} = K_{it}^{\alpha} N_{it}^{1-\alpha} \tag{51}$$

where $i \in [0,1]$ indexes the firms and t indexes time. To further simplify the analysis, I assume that each firm in the economy has the same prior distribution about R, W, D and

[2]Recently, L'Huillier and Phelan (2023) suggests that a model should amplify the response of prices to input price shocks and dampen the response of prices to demand shocks. Unfortunately, my model does the exact opposite. To dampen the response of prices to demand shocks, one may want to introduce information frictions that dampen the direct response of prices to demand shocks, and this needs to be strong enough to offset the amplification from quantities.

that the idiosyncratic noises that each firm faces are generated by the same distribution, i.e.,

$$\epsilon_{it}^{s} \sim N(0, \sigma_{\epsilon,s}^{2}) \tag{52}$$

for $s = r, w, d$ and $i \in [0, 1]$. As a result, all firms in this economy have the same degree of narrow framing, as measured by λ_r, λ_w and λ_d. Their responses to input-price shocks and demand shocks, as measured by $\alpha_s(\lambda_s)$ and $\beta_s(\lambda_s)$, are also identical across firms. In other words, the firms are different only because they have different realizations of these idiosyncratic noises $\epsilon_{it}^{r}, \epsilon_{it}^{w}, \epsilon_{it}^{d}$ in each period. In that case, the variance of idiosyncratic noises $\sigma_{\epsilon,s}^{2}$ not only represents the precision of idiosyncratic signals but also the cross-sectional distribution of idiosyncratic noises that each firm receives.

Firm-level Misoptimization With a Cobb-Douglas production function, we have

$$\frac{\gamma_k}{\gamma_{nk}} = \frac{\gamma_n}{\gamma_{kn}} = 1 < \epsilon$$

so that $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k > 0$ and $\gamma_{kn} - \frac{1}{\epsilon}\gamma_n > 0$. Therefore the following corollary is immediate from the previous propositions:

Corollary 1. *With the CRS Cobb-Douglas production function (51), the following hold:*

1. *The response of capital (labor) to a change in the wage rate (the rental rate of capital) is negative and dampened;*
2. *The response of capital (labor) to a change in the rental rate (the wage rate) is negative and dampened;*
3. *The response of capital and labor to a change in demand is positive and dampened;*
4. *The response of output to a change in input prices is negative and dampened;*
5. *The response of output to a change in demand is positive and dampened;*
6. *The response of price to a change in input prices is positive and dampened;*
7. *The response of price to a change in demand is positive and amplified.*

For each individual firm i, we can write out the optimal log-linearized decision rule as in (18) and (19), which I replicate here:

$$\begin{aligned} k_{it}(r_t, \tilde{w}_{it}, \tilde{d}_{it}) &= \alpha_r(\lambda_r) r_t + \alpha_w(\lambda_w)(w_t + \epsilon^w_{it}) + \alpha_d(\lambda_d)(d_t + \epsilon^d_{it}) \quad (53) \\ n_{it}(\tilde{r}_{it}, w_t, \tilde{d}_{it}) &= \beta_r(\lambda_r)(r_t + \epsilon^r_{it}) + \beta_w(\lambda_w) w_t + \beta_d(\lambda_d)(d_t + \epsilon^d_{it}) \quad (54) \end{aligned}$$

For each realization of (r_t, w_t, d_t), denote a rational firm's decision rule as $k^*(r_t, w_t, d_t)$ and $n^*(r_t, w_t, d_t)$. We know that for a rational firm, $\lambda_r = \lambda_d = \lambda_w = 1$, and hence

$$\begin{aligned} k^*(r_t, w_t, d_t) &= \alpha_r(1) r_t + \alpha_w(1) w_t + \alpha_d(1) d_t \quad (55) \\ n^*(r_t, w_t, d_t) &= \beta_r(1) r_t + \beta_w(1) w_t + \beta_d(1) d_t \quad (56) \end{aligned}$$

I define the misoptimization of firm i's input choices at period t as

$$\mathcal{M}^k_{it} \equiv k_{it}(r_t, \tilde{w}_{it}, \tilde{d}_{it}) - k^*(r_t, w_t, d_t) \quad (57)$$

and

$$\mathcal{M}^n_{it} \equiv n_{it}(\tilde{r}_{it}, w_t, \tilde{d}_{it}) - n^*(r_t, w_t, d_t) \quad (58)$$

The following proposition characterizes the properties of firm-level misoptimization:

Proposition 6 (Firm-level Misoptimization)**.** *Misoptimization of input choices $\mathcal{M}^k_{it}$ and $\mathcal{M}^n_{it}$ has the following properties:*

1. *$\mathcal{M}^k_{it}$ and $\mathcal{M}^n_{it}$ can be decomposed as*

$$\mathcal{M}^k_{it} = F^k(r_t, w_t, d_t) + G^k(\epsilon^r_{it}, \epsilon^w_{it}, \epsilon^d_{it}) \quad (59)$$

and

$$\mathcal{M}^n_{it} = F^n(r_t, w_t, d_t) + G^n(\epsilon^r_{it}, \epsilon^w_{it}, \epsilon^d_{it}) \quad (60)$$

where F^k, F^n are linear functions of (r_t, w_t, d_t) and G^k, G^n are linear functions of $(\epsilon^r_{it}, \epsilon^w_{it}, \epsilon^d_{it})$, defined by

$$\begin{aligned} F^k(r_t, w_t, d_t) &= (\alpha_r(\lambda_r) - \alpha_r(1)) r_t + (\alpha_w(\lambda_w) - \alpha_w(1)) w_t + (\alpha_d(\lambda_d) - \alpha_d(1)) d_t \quad (61) \\ F^n(r_t, w_t, d_t) &= (\beta_r(\lambda_r) - \beta_r(1)) r_t + (\beta_w(\lambda_w) - \beta_w(1)) w_t + (\beta_d(\lambda_d) - \beta_d(1)) d_t \quad (62) \\ G^k(\epsilon^r_{it}, \epsilon^w_{it}, \epsilon^d_{it}) &= \alpha_w(\lambda_w)\epsilon^w_{it} + \alpha_d(\lambda_d)\epsilon^d_{it} \quad (63) \\ G^n(\epsilon^r_{it}, \epsilon^w_{it}, \epsilon^d_{it}) &= \beta_r(\lambda_r)\epsilon^r_{it} + \beta_d(\lambda_d)\epsilon^d_{it} \quad (64) \end{aligned}$$

2. *Conditional on the realization of* (r_t, w_t, d_t)*, expected misoptimization is given by*

$$E[\mathcal{M}_{it}^k | r_t, w_t, d_t] = F^k(r_t, w_t, d_t) \tag{65}$$
$$E[\mathcal{M}_{it}^n | r_t, w_t, d_t] = F^n(r_t, w_t, d_t) \tag{66}$$

and F^k, F^n *are both increasing in* r_t *and* w_t *and decreasing in* d_t*.*

3. *Conditional on the realization of* (r_t, w_t, d_t)*, the variance of misoptimization is given by*

$$var[\mathcal{M}_{it}^k | r_t, w_t, d_t] = \alpha_w(\lambda_w)^2\sigma_{\epsilon,w}^2 + \alpha_d(\lambda_d)^2\sigma_{\epsilon,d}^2 \tag{67}$$
$$var[\mathcal{M}_{it}^n | r_t, w_t, d_t] = \beta_r(\lambda_w)^2\sigma_{\epsilon,r}^2 + \beta_d(\lambda_d)^2\sigma_{\epsilon,d}^2 \tag{68}$$

The conditional variance of capital misoptimization decreases with λ_w *if* $\lambda_w \geq \Lambda$ *and increases with* λ_w *if* $\lambda_w < \Lambda$*, where*

$$\Lambda \equiv \frac{\alpha(1-\alpha)(1-1/\epsilon)^2 + 1/\epsilon}{\alpha(1-\alpha)(1-1/\epsilon)^2 + 2/\epsilon} \tag{69}$$

Similarly, the conditional variance of labor misoptimization decreases with λ_r *if* $\lambda_r \geq \Lambda$ *and increases with* λ_r *if* $\lambda_r < \Lambda$*. Both conditional variances decrease with* λ_d *if* $\lambda_d \geq 1/2$ *and increases with* λ_d *if* $\lambda_d < 1/2$*.*

Part 1 of Proposition 6 immediately follows from the definition of misoptimization and the implications of Corollary 1: under the specification of the Cobb-Douglas production function (51), we have $\alpha_s(\lambda_s) \geq \alpha_s(1)$ and $\beta_s(\lambda_s) \geq \beta_s(1)$ for $s = r, w$, as well as $\alpha_d(\lambda_d) \leq \alpha_d(1)$, $\beta_d(\lambda_d) \leq \beta_d(1)$. It shows that the firm-level misoptimization can be decomposed into two parts: the first part, given by F^k and F^n, is a function of the true state (r_t, w_t, d_t) only and independent of the idiosyncratic noises received by the managers in the firm, while the second part, given by G^k and G^n, is solely a function of idiosyncratic noises and independent of the state (r_t, w_t, d_t). In other words, each firm's misoptimization can be partially predicted by the true state (r_t, w_t, d_t) and is therefore state-dependent.

Part 2 of Proposition 6 elaborates the state-dependence of misoptimization: on average, the misoptimization is large when input prices are high and demand is low. This is reminiscent of Flynn and Sastry (2020), where the authors argue that misoptimization is countercyclical due to countercyclical attention. The underlying mechanisms that generate the state dependence of misoptimization in my model and theirs are different. In their model, the state-dependence

of misoptimization results from both the cost structure and the fact that the firm internalizes the risk attitude of the household as the "tolerance for mistakes" when it makes output decisions; the latter channel delivers countercyclical misoptimization. In my model, this channel is muted; the state dependence of misoptimization comes solely from the fact that the firm's optimal responses to input prices and demand are attenuated by narrow framing. As a result, although my model is able to generate state-dependent misoptimization, the cyclicality of misoptimization remains inconclusive: when aggregate state is high, input prices tend to be high (which leads to larger misoptimization), and the demand tends to be high as well (which leads to smaller misoptimization).

Part 3 of Proposition 6 studies the properties of conditional variance of misoptimization. These conditional variances can be regarded as the degree of misoptimization resulting from the idiosyncratic noises, or the cross-sectional dispersion of misoptimization across firms. I find that these conditional variances present a non-monotonic relationship with the degree of narrow framing, λ_r, λ_w and λ_d. This is not surprising from the expression of (67) and (68). Consider the effect of increasing λ_w. Recall that a high λ_w stands for a smaller degree of narrow framing for the capital manager. On one hand, a smaller degree of narrow framing for the capital manager is associated with a higher signal precision $\sigma_{\epsilon,w}^{-2}$, which leads to a decrease in the degree of capital misoptimization (as the variance of idiosyncratic noise tends to be smaller). On the other hand, a smaller degree of narrow framing also implies a higher sensitivity to the idiosyncratic noise (i.e., $\alpha_w(\lambda_w)^2$ is increasing with λ_w), which amplifies the effect of idiosyncratic noise that the capital manager receives. These two forces affect the degree of misoptimization in opposite directions, and they offset each other at the point where $\lambda_w = \Lambda$. Hence, the degree of misoptimization is not monotonically decreasing with the degree of narrow framing: the largest degree of misoptimization is achieved when the degree of narrow framing is set to $\lambda_w = \Lambda$.

Narrow Framing and Misallocation Narrow framing can be regarded as a mechanism that generates misallocation. Since the firms make capital and labor choices with heterogeneous information, the capital and labor choices at the firm level are generally inconsistent with cost minimization under the complete-information, rational benchmark. This is in contrast to Flynn and Sastry (2020), where capital and labor choices are pinned down by minimizing the cost of producing the mistaken output required by the inattentive firm. In other words, at the firm level, my model can generate a wedge in the first-order condition for the capital-labor trade-off, which the model in Flynn and Sastry (2020) cannot. Define

this wedge as

$$\tau_{it} \equiv k_{it}(r_t, \tilde{w}_{it}, \tilde{d}_{it}) + r_t - n_{it}(\tilde{r}_{it}, w_t, \tilde{d}_{it}) - w_t \tag{70}$$

The following proposition analyzes the properties of τ_{it}.

Proposition 7 (Capital-labor Wedge)**.** *The capital-labor wedge τ_{it} can be written as*

$$\tau_{it} = (\alpha_r(\lambda_r) - \beta_r(\lambda_r) + 1)r_t + (\alpha_w(\lambda_w) - \beta_w(\lambda_w) - 1)w_t + \alpha_w(\lambda_w)\epsilon^w_{it} - \beta_r(\lambda_r)\epsilon^r_{it} \tag{71}$$

and has the following properties:

1. *τ_{it} is decreasing with both r_t and w_t;*
2. *the cross-sectional dispersion of τ_{it} is given by*

$$\alpha_w(\lambda_w)^2\sigma^2_{\epsilon,w} + \beta_r(\lambda_r)^2\sigma^2_{\epsilon,r} \tag{72}$$

and is decreasing with λ_w (or λ_r) when $\lambda_w \geq \Lambda$ (or $\lambda_r \geq \Lambda$), and increasing with λ_w (or λ_r) when $\lambda_w < \Lambda$ (or $\lambda_r < \Lambda$), where Λ is defined by (69).

We can see from (71) that the capital-labor wedge τ_{it} can be decomposed into two parts. The first part, $(\alpha_r(\lambda_r)-\beta_r(\lambda_r)+1)r_t+(\alpha_w(\lambda_w)-\beta_w(\lambda_w)-1)w_t$, is common across all firms, and the second part, $\alpha_w(\lambda_w)\epsilon^w_{it} - \beta_r(\lambda_r)\epsilon^r_{it}$, is idiosyncratic and generates cross-sectional dispersion. Again, the common part of the wedge results from the fact that narrow framing changes the responsiveness of input choices to changes in input prices. When r_t and w_t are high, τ_{it} tends to be more negative and generates larger distortions on average. The dispersion of capital-labor wedge comes solely from the idiosyncratic noises of the capital manager and labor manager in each firm. If there is perfect coordination within the firm, so that all the signals are precise, then $\tau_{it} = 0$ for all i and t. As we can see, narrow framing can endogenously generate state-dependent, dispersed capital-labor wedges, without the existence of any other distortions that are exogenous to the firm.

The endogenous, dispersed capital-labor wedges generated by narrow framing leads me to compare and contrast Equation (71) with the capital-specific distortions in Hsieh and Klenow (2009). I argue that although narrow framing generates similar dispersed capital-labor wedges, it is crucially different from the Hsieh-Klenow type of distortions. In their model, each individual firm faces an idiosyncratic, capital-specific distortion τ_{Kit}, so that the

trade-off between capital and labor, after log-linearization, is given by

$$\tau_{Kit} = -(k_{it} + r_t - w_t - n_{it}) \tag{73}$$

This type of distortion only places a wedge in the first-order condition for the capital-labor choice but does not change the elasticity of capital and labor choice to input price changes. Narrow framing, on the other hand, changes the elasticity of input choices to input prices, and as a result, it provides a richer structure on the capital-labor wedges: the distortion is endogenously state-dependent.

The second part of Proposition 7 establishes a non-monotonic relationship between the cross-sectional dispersion of the capital-labor wedge and the degree of narrow framing. Reducing the degree of narrow framing reduces the dispersion only when the degree of narrow framing is small enough, or equivalently, when the Kalman gain λ_i is large enough. The reason for this result follows exactly from my previous discussion on Part 3 of Proposition 6: when the information flow within the firm is highly frictional, improving the communication leads the firm to put more weight on its signals, which amplifies the responsiveness of firms to idiosyncratic noises and dominates the effect of improved communication. As a result, improving information flows and mitigating the degree of narrow framing within the firm do not necessarily reduce the dispersion of marginal products and improve allocative efficiency.

Lastly, I analyze the aggregate TFP losses due to narrow framing. To do that, I assume that aggregate output is given by

$$Y_t = \left(\int_0^1 (A_{it} Y_{it})^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{74}$$

where A_{it} denotes the firm-level productivity. I assume that the cross-sectional distribution of A_{it} is log-normal, i.e.,

$$\log A_{it} \sim N(0, \sigma_a^2) \tag{75}$$

and that A_{it} is independent of the idiosyncratic noises that the capital and labor managers receive.

Normalize the price of aggregate output to unity. A competitive aggregate good producer chooses Y_{it} to

$$\max Y_t - \int_0^1 P_{it} Y_{it} \, di \tag{76}$$

which yields the demand function for each firm i:

$$P_{it} = Y_t^{1/\epsilon} A_{it}^{1-1/\epsilon} Y_{it}^{-1/\epsilon} \tag{77}$$

Using our previous notation for the demand factor D in (3), define

$$D_{it} \equiv Y_t A_{it}^{\epsilon-1} \tag{78}$$

As before, I assume that each firm i receives a noisy signal about D_{it}, in the same form as (10) and (11).[3]

Define aggregate capital and labor as

$$K_t \equiv \int_0^1 K_{it}\, di$$

and

$$N_t \equiv \int_0^1 N_{it}\, di$$

If all firms are rational, the first-order conditions of capital and labor at the firm level imply that

$$K_{it} = \frac{A_{it}^{\epsilon-1}}{E[A_{it}^{\epsilon-1}]} K_t$$

and

$$N_{it} = \frac{A_{it}^{\epsilon-1}}{E[A_{it}^{\epsilon-1}]} N_t$$

and therefore aggregate total factor productivity A_t can be expressed as a generalized mean of firm-level productivity:

$$A_t \equiv \frac{Y_t}{K_t^{\alpha} N_t^{1-\alpha}} = E\left[A_{it}^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}} \tag{79}$$

Denote $A_t^* \equiv E\left[A_{it}^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}}$. Absent any distortions that impede the marginal products from equating across the firms, A_t^* can be regarded as the efficient level of aggregate TFP.

If the firms are subject to narrow framing, we can follow the same procedure: express the firm-level input choices K_{it} and N_{it} as a fraction of aggregate input K_t and N_t, and then plug them into the aggregation function (74) and calculate the aggregate TFP under narrow framing A_t. The tedious but simple algebra is relegated to Appendix A.3. The following result compares the aggregate TFP under narrow framing and the efficient benchmark A_t^*.

[3] In my model, from the firm's perspective, a demand shock is observationally equivalent to a productivity shock, and there is no way to distinguish them from each other.

Proposition 8 (TFP Loss From Narrow Framing)**.** *The aggregate total factor productivity of narrow-framing firms A_t satisfies*

$$A_t = A_t^* \times H(\lambda_r, \lambda_w, \lambda_d, \sigma^2_{\epsilon,r}, \sigma^2_{\epsilon,w}, \sigma^2_{\epsilon,d}, \sigma^2_a) \tag{80}$$

where $H(\cdot)$ is a function of Kalman gains, variances of idiosyncratic noises and the dispersion of idiosyncratic productivity, and

$$H(\lambda_r, \lambda_w, \lambda_d, \sigma^2_{\epsilon,r}, \sigma^2_{\epsilon,w}, \sigma^2_{\epsilon,d}, \sigma^2_a) < 1.$$

Furthermore, $H(\cdot)$ is decreasing in σ^2_a, increasing in λ_r and λ_w when $\lambda_r, \lambda_w \geq \Lambda$, and decreasing in λ_r and λ_w when $\lambda_r, \lambda_w < \Lambda$, where Λ is defined by (69).

Proposition 8 illustrates that narrow framing lowers the aggregate TFP, and the $H(\cdot)$ function can be regarded as a "productivity wedge" between narrow framing TFP and the efficient TFP. The fact that this wedge is smaller than unity is because aggregating log-normal signals leads the size of noise $\sigma^2_{\epsilon,r}, \sigma^2_{\epsilon,w}, \sigma^2_{\epsilon,d}$ to have a first-order effect on aggregate variables. As a result, it takes the same form as the papers in which the idiosyncratic variables follow log-normal distributions, such as David, Hopenhayn, and Venkateswaran (2016), David and Venkateswaran (2019), and Flynn and Sastry (2020). The analysis here is consistent with the takeaway from the misallocation literature (for example, Hsieh and Klenow (2009); Hopenhayn (2014); Restuccia and Rogerson (2017)): any mechanism that generates cross-sectional dispersion of marginal product at the firm level suppresses the aggregate TFP and lowers allocative efficiency. In my model, the mechanism is the idiosyncratic noisy signals received by the capital and labor manager due to narrow framing.

Proposition 8 also gives us the relationship between the productivity wedge $H(\cdot)$ and dispersion of idiosyncratic productivity σ^2_a, as well as the degree of narrow framing λ_r, λ_w. It is not surprising that the productivity wedge is wider if idiosyncratic productivity is more dispersed. Again, in my model, there is a non-monotonic relationship between the productivity wedge and degree of narrow framing: mitigating the degree of narrow framing reduces the TFP loss only when the information flow within the firm is adequately effective. This result is inherited directly from our previous non-monotonic results about degree of misoptimization (in Proposition 6) and dispersion of capital-labor wedge (in Proposition 7): the aggregate TFP loss in my model originates from the dispersion of misoptimization, or equivalently, the dispersion of marginal products of inputs, and they both have a non-monotonic relationship with the degree of narrow framing.

3 Narrow Framing in General Equilibrium

In this section, I study the general equilibrium implications of narrow framing by incorporating narrow-framing firms into an otherwise standard real business cycle (RBC) model. In particular, I am interested in how narrow framing would impact the impulse response functions of endogenous variables to exogenous macroeconomic shocks, such as productivity shocks, discount factor shocks and investment shocks.

For simplicity, I keep the assumption that all intermediate-good firms in the economy have the same degree of narrow framing, and that the idiosyncratic noises are generated from the same distribution (52).

3.1 The Model

Household The household sector is standard. A representative household owns capital, which is its only saving technology. It earns income by renting capital to firms, supplying labor to firms, and receiving dividends from all firms, and it consumes final goods.

The household enters each period with K_{t-1} units of capital. It chooses final-good consumption C_t, labor supply N_t and capital available at the beginning of next period K_t to

$$\max_{C_t,N_t,K_t} E\sum_{t=0}^{\infty}\beta^t Z_t\left(\log C_t - \frac{N_t^{1+\eta}}{1+\eta}\right) \tag{81}$$

subject to

$$C_t + Q_t(K_t - (1-\delta)K_{t-1}) = R_t K_{t-1} + W_t N_t + \Pi_t \tag{82}$$

where R_t is the rental rate of capital, W_t is the wage rate, and Π_t is the profit of firms. Z_t is an exogenous shock to discount factor: an increase in Z_t leads the household to place more weight on current utility than on future utility and hence makes the household less patient. Q_t is an exogenous investment shock, with steady state $\bar{Q} = 1$. An increase in Q_t means that capital becomes more expensive than final good consumption. Here, the price of the final good is normalized to 1.

The representative household's optimality conditions include a standard labor supply curve

and an Euler equation

$$\begin{aligned} C_t N_t^{\eta} &= W_t \\ Q_t Z_t / C_t &= \beta E_t \left[Z_{t+1}/C_{t+1}((1-\delta)Q_{t+1} + R_{t+1}) \right] \end{aligned}$$

which can be log-linearized around the steady state as follows (from now on, lower-case letters denote log deviations of the corresponding variables):

$$\begin{aligned} c_t + \eta n_t &= w_t \\ q_t + z_t - c_t &= E_t[z_{t+1} - c_{t+1}] + \beta \bar{R} E_t[r_{t+1}] + \beta(1-\delta) E_t[q_{t+1}] \end{aligned}$$

where $\bar{R}$ is the steady-state rental rate: $\bar{R} = 1/\beta - 1 + \delta$.

Final Good Producer The final good producer combines intermediate goods into final goods with a CES technology:

$$Y_t = A_t \left(\int_0^1 Y_{it}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{83}$$

where A_t is an exogenous productivity shock. This can be log-linearized into

$$y_t = a_t + \int_0^1 y_{it} \, di$$

Since the price of the final good is normalized to 1, the final good producer chooses Y_{it} to maximize its profit

$$\max Y_t - \int_0^1 P_{it} Y_{it}$$

subject to (83). The first-order condition defines the demand function of intermediate good i

$$P_{it} Y_{it}^{1/\epsilon} = A_t^{1-1/\epsilon} Y_t^{1/\epsilon} \tag{84}$$

Intermediate Good Producers Intermediate good producer i determines capital K_{it}, labor N_{it}, price P_{it} and output Y_{it} in two stages. In the first stage, the capital manager and the labor manager choose K_{it} and N_{it} to maximize

$$A_t^{1-1/\epsilon} Y_t^{1/\epsilon} \left(K_{it}^{\alpha} N_{it}^{1-\alpha} \right)^{1-1/\epsilon} - R_t K_{it} - W_t N_{it} \tag{85}$$

but with information frictions: the capital manager knows R_t but receives a noisy signal $\tilde{W}_{it}$ about W_t, the labor manager knows W_t but receives a noisy signal $\tilde{R}_{it}$ about R_t, and they both receive a common noisy signal $\tilde{D}_{it}$ about $D_t \equiv A_t^{\epsilon-1} Y_t$. In the second stage, the firm combines the capital and labor chosen under incomplete information into output Y_{it} by the production function (51), and sets the price according to the demand function (84).

The log-linearized optimal decision rule for the capital and labor manager of each firm is given by (18) and (19). Aggregating them for each realization of r_t, w_t, d_t gives the demand functions for capital and labor, (20) and (21). Substituting $d_t \equiv (\epsilon - 1)a_t + y_t$, we get

$$k_t^d = \alpha_r(\lambda_r) r_t + \alpha_w(\lambda_w) w_t + \alpha_d(\lambda_d)(y_t + (\epsilon - 1)a_t) \tag{86}$$

$$n_t^d = \beta_r(\lambda_r) r_t + \beta_w(\lambda_w) w_t + \beta_d(\lambda_d)(y_t + (\epsilon - 1)a_t) \tag{87}$$

where k_t^d and n_t^d denote the aggregate log-linearized demand of capital and labor respectively.

Market Clearing and Competitive Equilibrium Lastly, in a competitive equilibrium, the goods, labor, and capital markets clear, which entails the following log-linearized market clearing conditions

$$\begin{aligned} \bar{C} c_t + \bar{K}(k_t - (1-\delta)k_{t-1} + \delta q_t) &= \bar{Y} y_t \\ k_{t-1} &= k_t^d \\ n_t &= n_t^d \end{aligned}$$

In sum, given exogenous variables $\{z_t, q_t, a_t\}$, we end up with a system of linear equations that pins down endogenous variables $\{c_t, k_t, y_t, n_t, w_t, r_t\}$:

$$c_t + \eta n_t = w_t \tag{88}$$

$$q_t + z_t - c_t = E_t[z_{t+1} - c_{t+1}] + \beta \bar{R} E_t[r_{t+1}] + \beta(1-\delta) E_t[q_{t+1}] \tag{89}$$

$$\bar{Y} y_t = \bar{C} c_t + \bar{K}(k_t - (1-\delta)k_{t-1} + \delta q_t) \tag{90}$$

$$k_{t-1} = \alpha_r(\lambda_r) r_t + \alpha_w(\lambda_w) w_t + \alpha_d(\lambda_d)(y_t + (\epsilon - 1)a_t) \tag{91}$$

$$n_t = \beta_r(\lambda_r) r_t + \beta_w(\lambda_w) w_t + \beta_d(\lambda_d)(y_t + (\epsilon - 1)a_t) \tag{92}$$

$$y_t = a_t + \alpha k_{t-1} + (1-\alpha) n_t \tag{93}$$

Assume that $\{z_t, a_t, q_t\}$ are independent of each other and follow AR(1) processes:

$$z_t = \rho_z z_{t-1} + \epsilon_z \tag{94}$$

$$a_t = \rho_a a_{t-1} + \epsilon_a \tag{95}$$

$$q_t = \rho_q q_{t-1} + \epsilon_q \tag{96}$$

where $\epsilon_z \sim N(0, \sigma_z^2)$, $\epsilon_a \sim N(0, \sigma_a^2)$, $\epsilon_q \sim N(0, \epsilon_q^2)$ are i.i.d. and independent of each other.

3.2 Quantitative Results

The above system of linear equations can be solved with standard methods once the deep parameters are specified. In this subsection, I present the impulse response functions of endogenous variables for production sectors with different degrees of narrow framing, for a particular set of deep parameters. Before showing the results, I should point out that the "quantitative" exercise that follows should be read qualitatively, since I simply pick these deep parameters from the range of values that the literature typically uses. The purpose of this exercise is to give us some insights into how narrow framing would (qualitatively) change the general equilibrium behaviors of endogenous variables, instead of seriously quantifying the effects of narrow framing by calibration or estimation. I leave calibration and estimation to future work.

The parameters used to generate the IRFs are summarized in Table 1.

Parameter	Value	Description
β	0.99	Household's Discount Factor
α	0.3	Capital Share of Income
δ	0.025	Capital Depreciation Rate
ϵ	6	Elasticity of Substitution
η	3	Inverse Frisch Elasticity of Labor
ρ_z	0.5	Persistence of Discount Factor Shock
ρ_a	0.9	Persistence of Productivity Shock
ρ_q	0.5	Persistence of Investment-specific Shock
σ_z	0.005	Std. Dev. of Discount Factor Shock
σ_a	0.007	Std. Dev. of Productivity Shock
σ_q	0.005	Std. Dev. of Investment-specific Shock

Table 1: Choice of Parameters

In Table 1, I leave out three information parameters that represent the degree of narrow

framing, namely, the Kalman gains λ_r, λ_w and λ_d. In the analysis that follows, in order to investigate the impact of each information parameter on the impulse responses of endogenous variables, each time I fix two of them at unity (which is the value at the rational benchmark) and vary the remaining one within the interval $[0, 1]$.

3.2.1 IRFs to a Productivity Shock

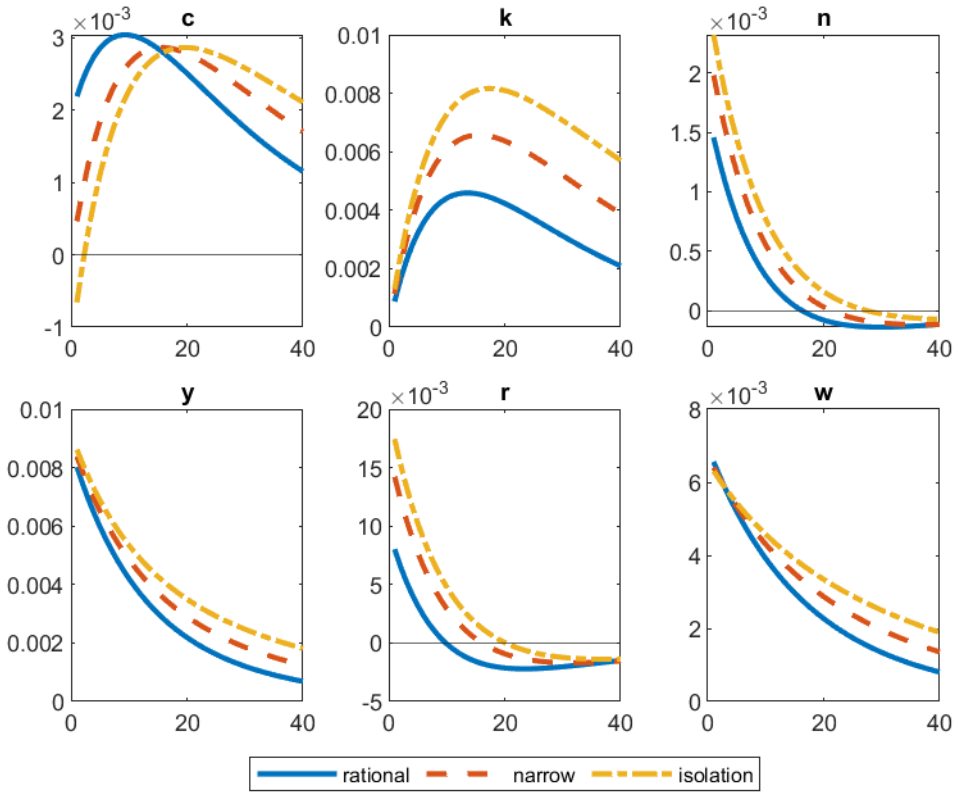
I first study the impact of narrow framing on the impulse responses of endogenous variables to a productivity shock. Figure 1, Figure 6 and Figure 7 illustrate IRFs to a one-standard-deviation TFP shock for production sectors with different degrees of narrow framing. In Figure 1, I fix $\lambda_r = \lambda_d = 1$ and incorporate only the capital manager's noisy signal about wage rate. The blue, solid line corresponds to the IRFs of a rational firm, or, in other words, the case where $\lambda_w = 1$. The red, dashed line corresponds to the IRFs of a narrow-framing firm with $\lambda_w = 0.5$. The yellow, dash-dotted line corresponds to the IRFs of a firm with an isolated capital manager, who receives a completely uninformative signal about wage rate, so that $\lambda_w = 0$. Similarly, in Figure 6, I fix $\lambda_w = \lambda_d = 1$ and plot the IRFs for $\lambda_r = 1$, $\lambda_r = 0.5$ and $\lambda_r = 0$, and in Figure 7, I fix $\lambda_r = \lambda_w = 1$ and plot the IRFs for $\lambda_d = 1$, $\lambda_d = 0.5$ and $\lambda_d = 0$.

The Effect of Noisy Wage Signal From Figure 1, we can see that incorporating the capital manager's uncertainty about wage rate amplifies the responses of capital, labor, output and rental rate of capital to a productivity shock. The responses of consumption and wage are dampened initially but amplified subsequently. Quantitatively, the impact of the capital manager's uncertainty about the wage is large. The initial response of rental rate can be amplified by up to 100% and that of labor can be amplified by up to 67%. The impulse of consumption is significantly dampened by narrow framing and the sign of the response can even be reversed when the wage signal is sufficiently noisy for the capital manager. The impact of noisy wage signals on the responses of capital, output and wage is initially weak but becomes more pronounced subsequently.[4] In particular, the noisy wage signal generates an amplified and more persistent hump-shaped IRF for capital: the peak response of capital

[4]Personally, I find this "initial underreaction, subsequent overreaction" pattern for consumption intriguing, since this is consistent with the findings in the MPC literature: aggregate consumption underreacts to current income shocks and overreacts to past income shocks (Hall, 1978; Carroll et al., 1992). Many theoretical models can explain this phenomenon (for instance, Carroll (1997); **?**); Luo (2008)), but all these models introduce frictions on the household side. Here, I show that frictions on the firm side alone can also generate such behavior through general equilibrium feedback to households.

for the firm with an isolated capital manager is almost twice as large as that of the rational benchmark, and meanwhile, the arrival of the peak response is delayed as the wage signal becomes noisier.

Figure 1: IRFs: Productivity Shock, Noisy Wage Signal Only



Note: λ_r and λ_d are set to 1. The "rational" line corresponds to the case where $\lambda_w = 1$. The "narrow" line corresponds to the case where $\lambda_w = 0.5$. The "isolation" line corresponds to the case where $\lambda_w = 0$.

The results in Figure 1 can be interpreted graphically, using the basic supply-demand analysis on today and tomorrow's capital and labor markets[5]. Combining (91), (92) and (93), we get the demand curves for capital and labor in each period:

$$k_t^d(r_t; w_t, n_t, a_t) = \frac{(1-\alpha)\lambda_d}{1-\lambda_d\alpha} n_t + \frac{\alpha_r(\lambda_r)}{1-\lambda_d\alpha} r_t + \frac{\alpha_w(\lambda_w)}{1-\lambda_d\alpha} w_t + \frac{\epsilon\lambda_d}{1-\alpha\lambda_d} a_t$$

$$n_t^d(w_t; r_t, k_{t-1}, a_t) = \frac{\alpha\lambda_d}{1-(1-\alpha)\lambda_d} k_{t-1} + \frac{\beta_r(\lambda_r)}{1-(1-\alpha)\lambda_d} r_t + \frac{\beta_w(\lambda_w)}{1-(1-\alpha)\lambda_d} w_t + \frac{\epsilon\lambda_d}{1-(1-\alpha)\lambda_d} a_t$$

[5]By today I mean the time period when the shock hits the economy (denote it as time t), and by tomorrow I mean the time period following the initial shock (denote it as time $t+1$). I will use today and time t, tomorrow and time $t+1$ interchangeably.

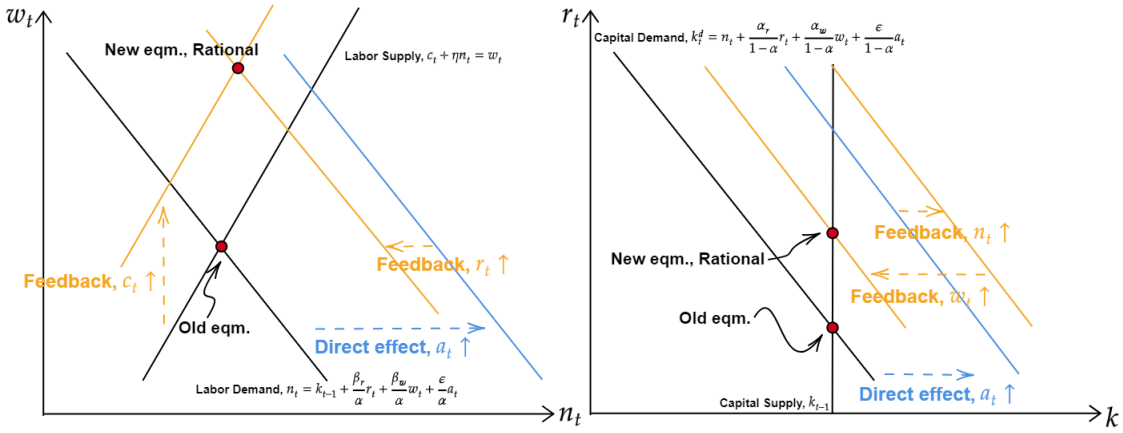
The labor supply curve is given by (88), and the capital supply curve is predetermined by k_{t-1}, the capital stock that the household chose in the preceding period. In the analysis that follows, I plot these supply and demand curves on a supply-demand diagram and provide a qualitative explanation for the patterns in Figure 1.

First, I look into today's labor market, which pins down the initial response w_t and n_t. In the rational benchmark, as illustrated in the left panel of Figure 2, the productivity shock leads the firm to increase labor n_t and wage w_t. The new equilibrium results from the movements on both the demand and supply side of labor. On the demand side, an increase in a_t introduces a direct right shift of the labor demand curve since labor is more productive. At the same time, a_t also shifts the capital demand curve to the right and hence pushes up rental rate r_t, which introduces an offsetting effect on labor demand. On the supply side, since a positive a_t shock increases y_t and w_t, it leads the household to choose a higher c_t (as the result of a positive income effect and a positive substitution effect), which shifts the labor supply curve inwards.

Incorporating the capital manager's noisy signal about wage rate changes the above analysis, which is analyzed graphically in Figure 3. Letting $\lambda_w < 1$ makes the labor demand curve flatter. As a result, for a productivity shock of the same size, the increases in w_t and n_t due to the direct effect are dampened. On the supply side, this implies a dampened indirect response from increases in c_t, and hence the labor supply curve shifts inward by a smaller magnitude than the rational benchmark. On the demand side, the increase of r_t in the capital market is also amplified due to a dampened response in w_t (which I will discuss later), hence the indirect response from increases in r_t is also amplified; the labor demand curve would shift left from the direct effect by a slightly larger magnitude. The left panel of Figure 3 illustrates that it is possible for the new equilibrium under narrow framing to amplify the response of labor and attenuate the response of wage, as the IRFs in Figure 1 indicate.

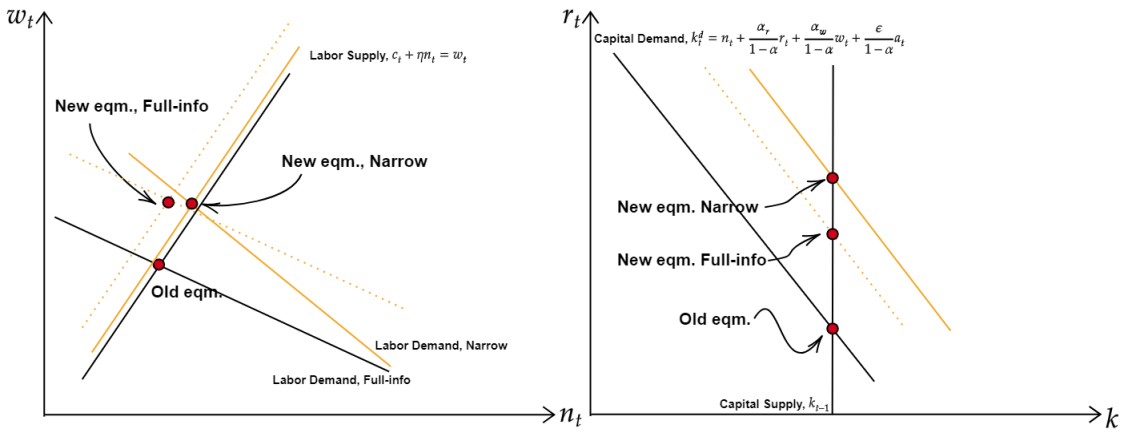
The analysis on today's capital market is relatively simple. Since the capital supply k_{t-1} is predetermined and does not shift with shocks in time t, the shocks will only lead to changes in today's rental rate r_t. As we can see from the right panel of Figure 2, a positive productivity shock impacts the demand for capital via two channels. On one hand, a positive productivity shock increases marginal product of capital and directly shifts the capital demand curve to the right, as the blue solid curve illustrates. On the other hand, a positive productivity shock leads to a higher wage rate w_t and a higher employment n_t, which introduces two offsetting effects on the capital demand; in Figure 2, I illustrate the case where the negative indirect effect of w_t dominates the positive indirect effect of n_t on capital demand. Therefore, it is

Figure 2: Impacts of Productivity Shock on Today's Factor Market: Mechanism



Note: Here $\lambda_r = \lambda_w = \lambda_d = 1$. The left panel delineates the demand and supply curves for today's labor market, and the right panel for today's capital market (note that the capital supply curve in each period is predetermined by household). On impact, a positive shock on a_t shifts labor demand curve and capital demand curve to the right (as the blue arrows and curves show), hence $w_t \uparrow$, $n_t \uparrow$ and $r_t \uparrow$. As shown in the right panel, $w_t \uparrow$ shifts capital demand inward, and $n_t \uparrow$ shifts capital capital demand outward; on the picture, I illustrate the case where the former force dominates the latter. On the left panel, $r_t \uparrow$ shifts capital demand curve inward; in response to $w_t \uparrow$ and $y_t \uparrow$, $c_t \uparrow$, hence labor supply curve will shift upward. The positions of the old and new equilibrium in today's labor and capital market are highlighted in the diagrams by red dots.

Figure 3: Impacts of Productivity Shock on Today's Factor Market: Narrow Framing



Note: The dotted lines are inherited from Figure 2 and illustrate the equilibrium response of a rational firm. The solid lines illustrate the equilibrium response of a narrow-framing firm with $\lambda_r = \lambda_d = 1$ and $\lambda_w < 1$.

clear that the capital manager's noisy wage signal amplifies the increase of r_t, because the dampened indirect effect of w_t works in the opposite direction from the direct effect and is dominated by the latter, as shown on the right panel of Figure 3. Note that from our previous discussion, the increase in n_t is amplified in the narrow-framing equilibrium, which amplifies the positive indirect response of capital demand. This force will lead the rental rate r_t to increase even more and thus further amplify the response of rental rate r_t to a productivity shock. This explains the large amplification effect of the capital manager's noisy wage signal on rental rate in Figure 1.

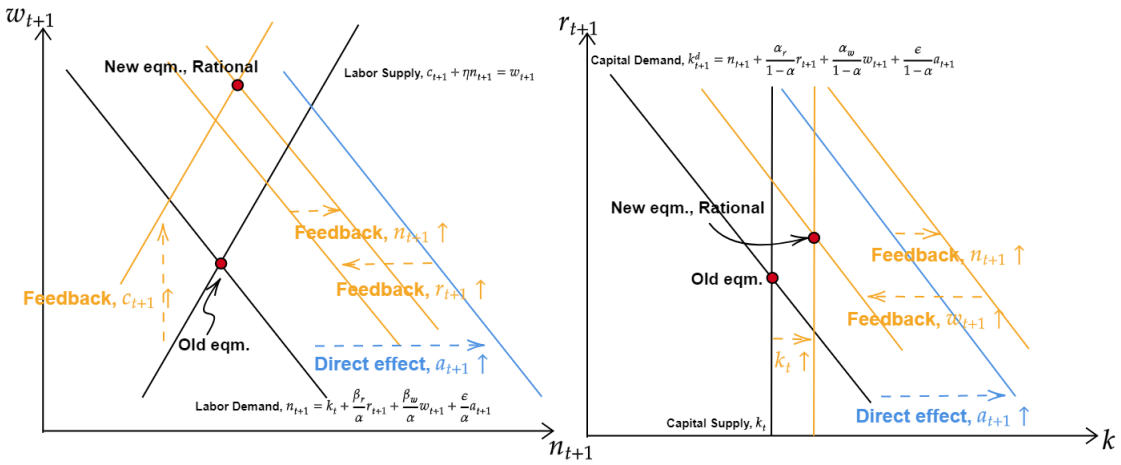
Following the above discussions on today's factor markets, it is immediate that the response of y_t will be amplified by the noisy wage signal, since the response of labor n_t is amplified. The analysis of consumption c_t and capital k_t is slightly more involved. Consumption c_t can be affected by a productivity shock via three channels: first, a positive productivity shock boosts income y_t and hence introduces a positive income effect on c_t; second, a positive productivity shock increases wage w_t, which introduces a positive intratemporal substitution effect on consumption c_t; lastly, since the productivity shock is persistent, the household would expect the future return to capital r_{t+1} to rise, and this introduces a negative intertemporal substitution effect on c_t via Euler equation (89). The capital manager's noisy wage signal amplifies the income effect channel, dampens the intratemporal channel, and amplifies the intertemporal channel (because the increase in r_{t+1} is amplified, which will be

discussed later). If the amplification of intertemporal channel is strong enough, the response of c_t will be dampened and may even reverse sign, which is the case when the wage signal is uninformative, i.e., $\lambda_w = 0$. Since the increase in income y_t is amplified, the capital available for next period (or equivalently, the saving of household), k_t, has to increase, and the increase has to be amplified due to a dampened response of c_t.

Lastly, I explain the reason for the subsequent overreaction of consumption and wage dynamics. To do this, I need to first analyze tomorrow's labor and capital market. The analysis is almost the same as what I did above for today's factor markets, with one exception: in period $t+1$, the capital supply curve, given by k_t, shifts to the right due to the positive income effect in period t. Figure 4 shows how it would change the supply-demand analysis of tomorrow's factor markets, absent any information friction. Apart from the feedback effect from the increases in r_{t+1}, the increase in k_t introduces another feedback effect that is in the opposite direction to the one generated by the increase in r_{t+1}. On the left panel of Figure 4, I plot the case where the feedback effect by k_t dominates that by r_{t+1}, so that overall the feedback effect is in the same direction as the direct effect. For tomorrow's capital market, the right shift of capital supply curve increases r_{t+1} by less than the case of today's capital market.

With the noisy wage signal, the above analysis is changed, as illustrated in Figure 5. Again, the noisy wage signal flattens the labor demand curve, and generates a smaller increase in wage w_{t+1} and labor n_{t+1}. A dampened wage increase attenuates the feedback effect by consumption c_{t+1} and hence the labor supply curve will not move as much as in the rational case. On the demand side of labor, if the overall feedback effect is dominated by the one generated by k_t, then the amplified response of k_t due to the noisy wage signal leads to an amplified right-shift of labor demand. As in the left panel of Figure 5, it is possible that this amplification effect is strong enough to let the final labor demand curve cross the final labor supply curve at a point northeast of the rational equilibrium, in which case the overall response of wage w_{t+1} is amplified. This is consistent with what we observe in Figure 1: overreaction of wage occurs in the periods when capital is significantly amplified. The more capital is amplified, the larger the wage overreaction. On the right panel of Figure 5, I plot the case where the response of r_{t+1} is amplified. However, one could imagine that if the increase in capital supply is significantly amplified, the response of rental rate r_{t+1} may be dampened or even reverse sign: it is possible that the new, narrow-framing equilibrium rental rate falls below the old equilibrium. This is exactly what we see in Figure 1: the rental rate falls below zero at some point. When this happens, the intertemporal substitution effect increases consumption since the interest rate is lower. Further amplifying k_t leads to an amplification

Figure 4: Impacts of Productivity Shock on Tomorrow's Factor Market: Mechanism



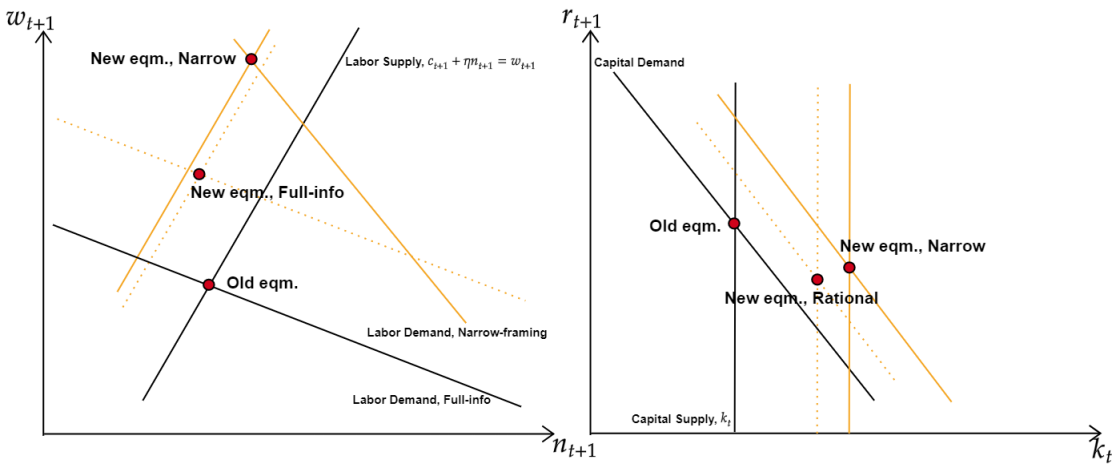
Note: Here $\lambda_r = \lambda_w = \lambda_d = 1$. Since the productivity shock is persistent, the direct effect on labor and capital demand is the same as in Figure 2. The only difference is that $y_t \uparrow \implies k_t \uparrow$, which shifts capital supply curve tomorrow rightward. $k_t \uparrow$ introduces an additional positive feedback effect on labor demand tomorrow; the left panel illustrates the case where the feedback effect from $k_t \uparrow$ dominates the one from $r_{t+1} \uparrow$.

in the drop of rental rate, which in turn amplifies the intertemporal substitution effect on consumption. As a result, the noisy wage signal amplifies consumption in periods when capital is significantly amplified.

The Effect of Noisy Rental Rate Signal Figure 6 plots the IRFs when the only friction is the labor manager's noisy signal about rental rate of capital. The first observation is that the responses of wage and labor are amplified by the noisy rental rate signal. From the above discussions, this is not surprising: since the response of labor demand to rental rate changes is dampened, and the rental rate increase works in the opposite direction to and is dominated by the direct effect of productivity shock, the initial responses of both wage and labor are amplified.

We can see that compared with noisy wage signals, the impact of noisy rental rate signals on IRFs of endogenous variables to a productivity shock is not as salient. The impulse response functions of consumption, capital, output and rental rate of capital are almost identical, no matter how informative the labor manager's signal is. This is due to the structure of our RBC model. In the case of noisy wage signals, the reason why narrow framing makes a

Figure 5: Impacts of Productivity Shock on Tomorrow's Factor Market: Narrow Framing



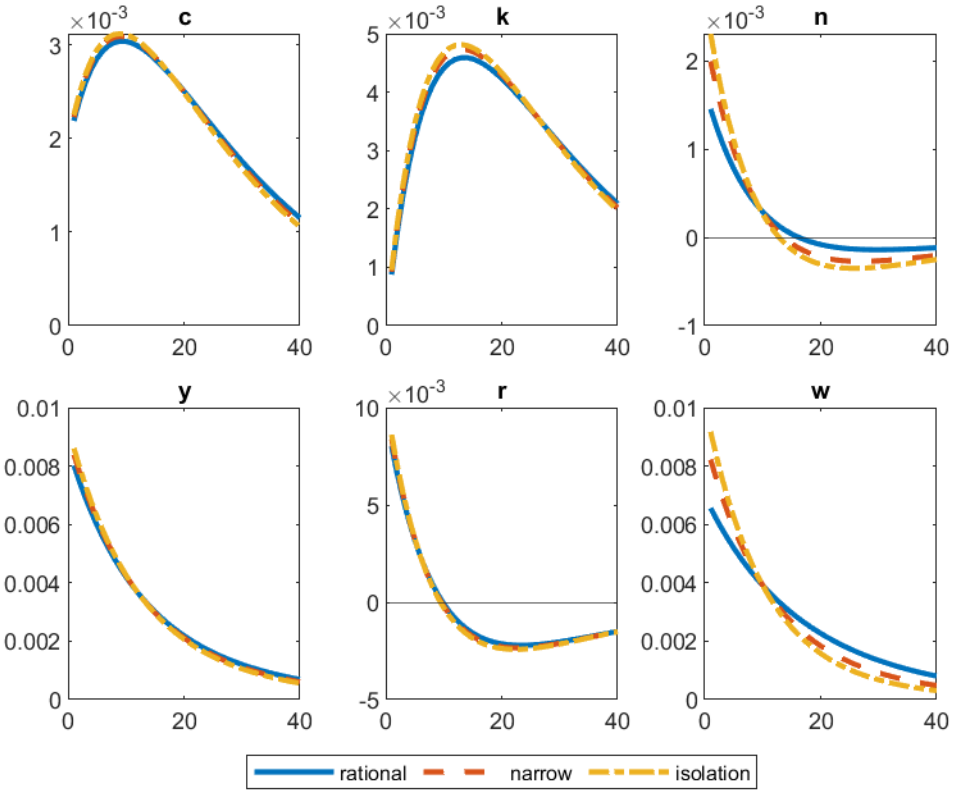
Note: The dotted lines inherit from Figure 4 and illustrate the equilibrium response of a rational firm. The solid lines illustrate the equilibrium response of a narrow-framing firm with $\lambda_r = \lambda_d = 1$ and $\lambda_w < 1$. Both panels omit the intermediate steps and only plot the final demand and supply curve for capital and labor (the yellow, solid lines) for the narrow-framing equilibrium (i.e., the case where $\lambda_w < 1$).

salient difference from the rational benchmark is that the rental/interest rate is significantly amplified by narrow framing, which generates a large dampening effect on the response of consumption and a large amplification effect on the response of capital. In contrast, what noisy signals on the rental rate do is dampen the direct response of rental rate of capital r_t to a productivity shock. However, since the capital demand function shifts one-for-one with labor n_t, the feedback from the amplified responses of labor largely offsets the dampened rental rate response brought by narrow framing of the labor manager. The same logic can be applied to tomorrow's capital market as well. As a result, the noisy signal on the rental rate cannot effectively change the behavior of the interest rate, which limits its ability to affect household behavior.

The Effect of Noisy Demand Signal Figure 7 plots IRFs when the only friction is the noisy signal about firm demand. We can see that the responses of capital, labor, rental rate and wage are significantly dampened. The initial response of consumption is amplified, but the subsequent response is dampened. The output response is dampened modestly.

Basically, what the noisy signal about firm demand does is mitigate the direct effect from a

Figure 6: IRFs: Productivity Shock, Noisy Rental Rate Signal Only



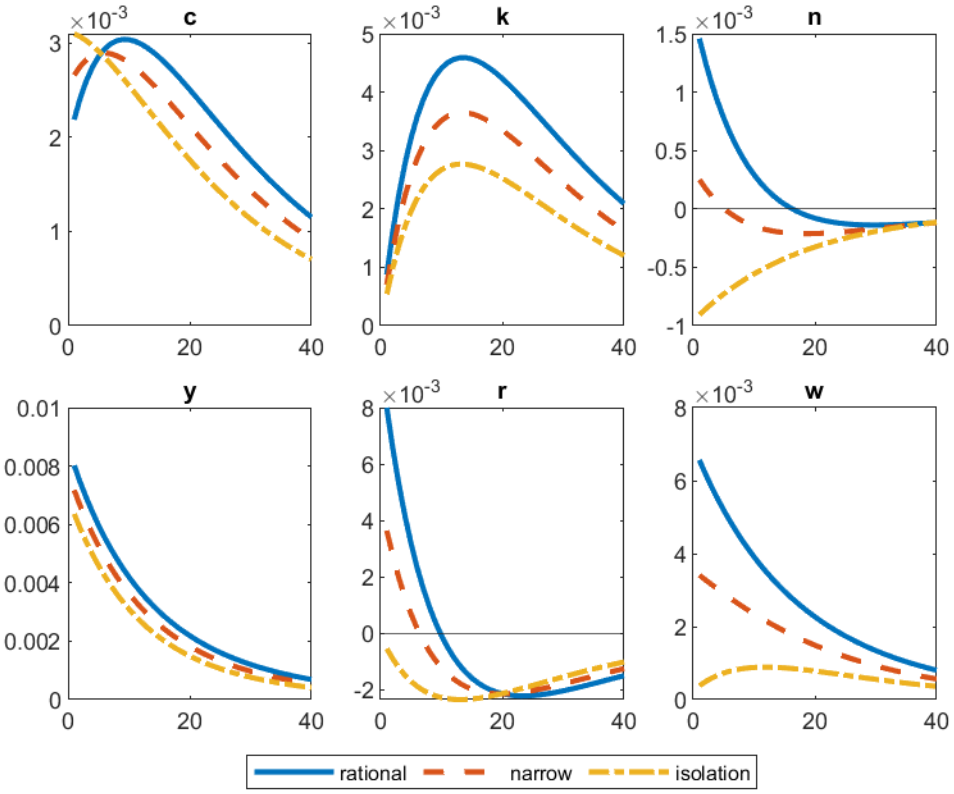
Note: λ_w and λ_d are set to 1. The "rational" line corresponds to the case where $\lambda_r = 1$. The "narrow" line corresponds to the case where $\lambda_r = 0.5$. The "isolation" line corresponds to the case where $\lambda_r = 0$.

productivity shock: it reduces the magnitude of right shifts for capital and labor demand that result from the boosted marginal products, and nothing else. In particular, it does not change the slope of demand curves and responsiveness of feedback effects, which is strikingly different from the previous two cases. This can be most easily seen from the dynamics of consumption: here, the initial response of consumption is amplified exactly because the increase in expected future interest rate is dampened. The attenuated interest rate increase causes a dampened decrease in consumption, and the overall effect on consumption is amplified. In contrast, recall that in the case of noisy wage signals, the initial consumption is dampened because the interest rate response is amplified, which enlarges the negative, offsetting effects on consumption that result from the Euler equation.

In the extreme case of isolation, where $\lambda_d = 0$, the IRFs in this case are strikingly different from other cases and therefore require an extra discussion. When $\lambda_d = 0$, the firm behaves as if it does not even realize that there is a productivity shock (even though the shock does hit). As a result, the business cycle is solely determined by the response of the household: in response to the income boom brought by the productivity shock, the household increases consumption and capital. As we discussed before, the increase in consumption will shift the labor supply curve up due to the intratemporal substitution between consumption and leisure. However, the firm is completely ignorant of the shock and therefore labor and capital demand remain unchanged. Hence wage will increase and labor will decrease as a result of the upward shift of labor supply curve. The feedback effect of wage increase passes through to the capital demand and lowers rental rate r_t.

Summary In sum, I conclude that narrow framing can significantly change the impact and propagation of a productivity shock in an otherwise standard RBC model. However, the ways in which the three idiosyncratic signals change the impact and propagation of a productivity shock are different. Roughly speaking, the capital manager's noisy wage signal and the labor manager's noisy rental rate signal are "amplifying" frictions in the sense that they amplify the response of output to a productivity shock, as well as the responses of capital, interest/rental rate and wage. The noisy signal about firm demand is, on the other hand, a "dampening" friction, in the sense that it attenuates the response of output to a productivity shock, as well as the responses of capital, interest/rental rate and wage. Of the two amplifying frictions, the IRFs generated by the capital manager's noisy wage signal are more saliently different from the rational benchmark than the ones generated by the labor manager's noisy rental rate signal.

Figure 7: IRFs: Productivity Shock, Noisy Demand Signal Only



Note: λ_w and λ_r are set to 1. The "rational" line corresponds to the case where $\lambda_d = 1$. The "narrow" line corresponds to the case where $\lambda_d = 0.5$. The "isolation" line corresponds to the case where $\lambda_d = 0$.

3.2.2 IRFs to an Investment Shock

I now study the impact of narrow framing on IRFs of endogenous variables to a one-standard-deviation investment shock on q_t. The key difference between an investment shock and a productivity shock is that in my model, a productivity shock is a supply-side shock in the sense that it impacts the firm directly (by increasing the marginal products of labor and capital and hence shifting the labor and capital demand curves), while an investment shock is a demand-side shock. Since in my model it is the household who makes the capital accumulation/investment decisions, an investment shock in the form of an increase in the price of capital q_t impacts the household decisions directly, the effect of which passes through to the firms indirectly via general equilibrium.

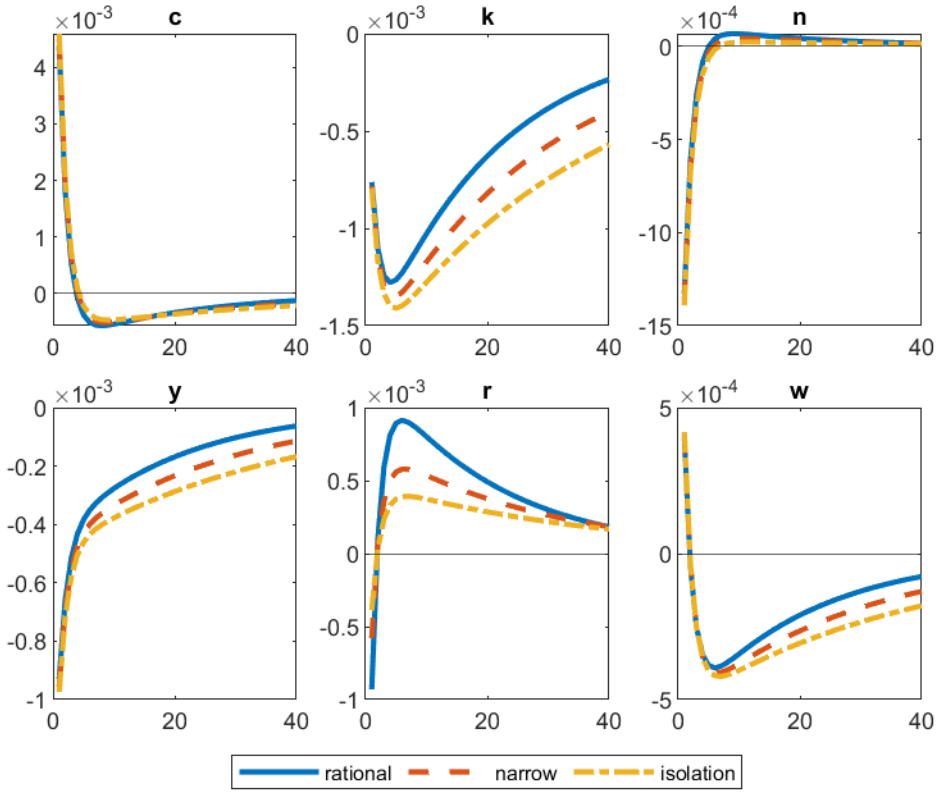
As before, I introduce one information friction at a time and see how each of these noisy signals affects the behaviors of endogenous variables in general equilibrium.

The Effect of Noisy Wage Signal As before, I first mute the information friction on rental rate of capital and firm demand, and only incorporate the noisy signal on wage rate for capital managers. In Figure 8, I plot the IRFs of endogenous variables to an investment shock for firms with different degrees of informativeness about wage. We can see that the noisy wage signal amplifies the response of capital, wage and output and dampens the response of rental rate. For output, capital and wage, the amplification effect becomes more salient after about 5 periods. The responses of labor and consumption are slightly amplified initially and dampened subsequently.

Again, I can interpret the IRFs in Figure 8 with basic supply-demand analysis. Before that, I summarize the direct effect of an investment shock on household's behavior. The direct effect of an increase in q_t is twofold. On one hand, it lowers the effective real interest rate and hence increases current consumption c_t, lowers future consumption c_{t+1} and capital k_t, according to the Euler equation (89). On the other hand, according to (90), it reduces the resources available, leading to a drop in c_t and k_t. In the quantitative exercise, the Euler-equation channel dominates, and hence the direct effect of q_t leads to an increase in c_t and a decrease in k_t.

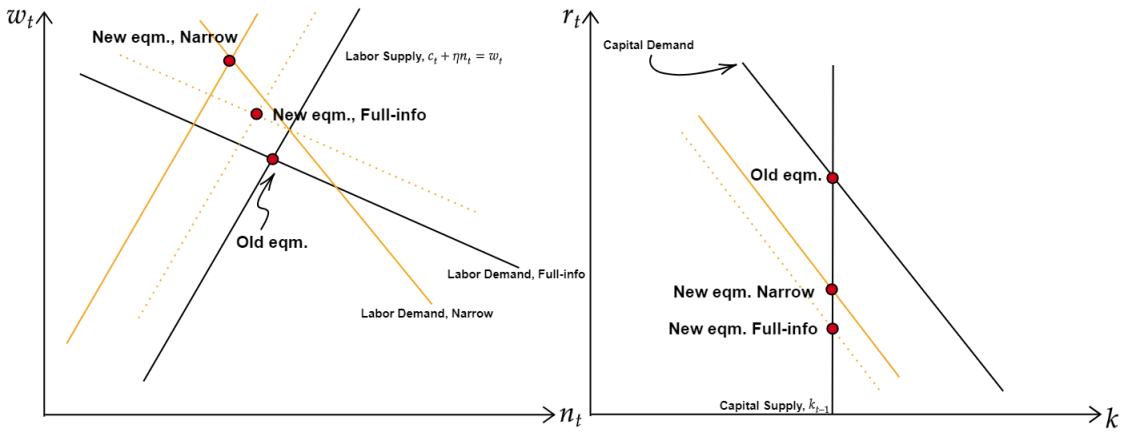
Now I study how the direct impact of an investment shock propagates to the firms. Figure 9 illustrates the propagation mechanism on today's labor and capital market. On the left panel, we can see that an increase in consumption shifts the labor supply curve inward. This

Figure 8: IRFs: Investment Shock, Noisy Wage Signal Only



Note: λ_d and λ_r are set to 1. The "rational" line corresponds to the case where $\lambda_w = 1$. The "narrow" line corresponds to the case where $\lambda_w = 0.5$. The "isolation" line corresponds to the case where $\lambda_w = 0$.

Figure 9: Impacts of Investment Shock on Today's Factor Market: Narrow Framing



Note: Dotted lines illustrate the shifts of supply and demand curves for a rational firm. Solid lines illustrate the shifts of supply and demand curves for a narrow-framing firm with $\lambda_w < 1$.

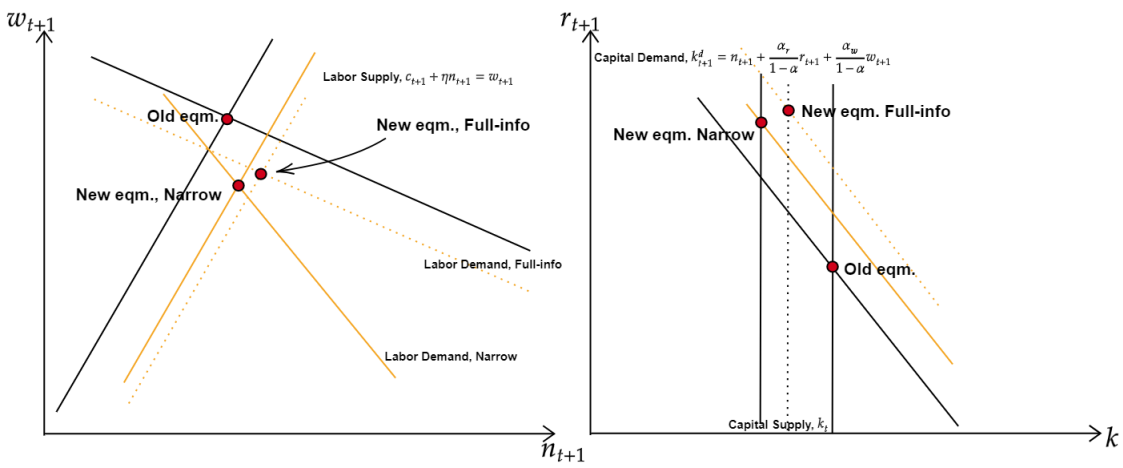
lowers labor n_t and generates an upward pressure on wage w_t, which further introduces a positive intratemporal substitution effect on consumption, shifting the labor supply curve further to the left. The increase in wage w_t and decrease in labor n_t both negatively affect capital demand and hence shift the capital demand curve downwards on the right panel of Figure 9. As a result, r_t decreases. This introduces a positive feedback effect on labor demand, and hence shifts the labor demand curve rightwards. As we discussed before, narrow framing of the capital manager dampens the responsiveness of labor and capital demand to changes in wage. It reduces the downward shift of capital demand in the capital market, and hence in a narrow-framing equilibrium, interest rate r_t falls by less compared to the rational benchmark, as illustrated in the right panel of Figure 9. This explains why the initial response of interest rate is dampened in Figure 8. A dampened decrease in r_t leads to a dampened right-shift of labor demand curve on the left panel of Figure 9. With the amplified response of current consumption under narrow framing (which I will discuss later), it is possible for the firm to end up having an amplified decrease in labor n_t and amplified increase in wage w_t, as illustrated in the left panel of Figure 9. This reproduces the IRFs of wage and labor in Figure 8.

Regarding tomorrow's labor and capital market, the supply-demand analysis is similar and delineated in Figure 10. The direct impact of the investment shock lowers tomorrow's consumption c_{t+1}, hence shifts the labor supply curve downward. This leads to an increase in

tomorrow's labor n_{t+1} and a decrease in tomorrow's wage w_{t+1}, which generates a positive feedback effect on capital demand and shifts the capital demand curve rightward. Recall that the direct impact of investment shock lowers the capital supply k_t, which shifts the capital supply curve to the left. As a result, interest rate r_{t+1} increases. The negative feedback of this interest rate increase, together with the decrease in k_t, shifts the labor demand curve inwards. Moreover, the increase in interest rate r_{t+1} introduces a feedback effect to today's consumption c_t, tomorrow's consumption c_{t+1} and capital k_t via the Euler equation: since the surging real interest rate makes saving more attractive, it generates a negative effect on c_t and a positive effect on c_{t+1} and k_t. Now, with narrow framing of the capital manager, the right-shift of capital demand curve is dampened because the capital manager's responsiveness to the wage rate is attenuated. Also, the amplified decrease in n_t leads to an amplified decrease in output y_t and causes the capital k_t to decrease even more, and therefore the decrease in capital supply k_t is amplified. The right panel of Figure 10 shows that it is possible for the increase in interest rate r_{t+1} to be dampened as in Figure 8. As a result, the positive feedback of the increase in r_{t+1} on capital k_t is also dampened and hence the overall negative effect on capital k_t is further amplified. The amplified decrease in capital and the dampened increase in interest rate can amplify the downward shift of labor demand curve and generate a dampened increase in labor n_{t+1} and an amplified decrease in wage w_{t+1}.

Lastly, I explain the amplification of today's consumption c_t. There are three channels that provide general equilibrium feedback to current consumption from the supply side. First, the increase in wage rate w_t generates a positive feedback effect on c_t via intratemporal substitution between leisure and consumption. Second, the decrease in output y_t generates a negative feedback effect on c_t via the resource constraint. Lastly, the increase in future rental rate r_{t+1} generates a negative feedback effect on c_t via the Euler equation. With narrow framing of the capital manager, the positive intratemporal-substitution channel is amplified, the negative resource-constraint channel is amplified, and the negative Euler-equation channel is dampened. Hence, the overall effect depends on the relative strength of these channels. If the amplification effect generated by the amplifying, positive intratemporal-substitution channel and the dampening, negative Euler-equation channel dominates the amplifying, negative resource-constraint channel, then consumption c_t is amplified, which is the case in the IRF of consumption in Figure 8. As we can also see in Figure 8, since the influences of narrow framing on the general equilibrium feedback effects offset each other, the net impact of narrow framing on the general equilibrium feedback to consumption is small, and the IRFs are almost identical to each other for different types of firms.

Figure 10: Impacts of Investment Shock on Tomorrow's Factor Market: Narrow Framing

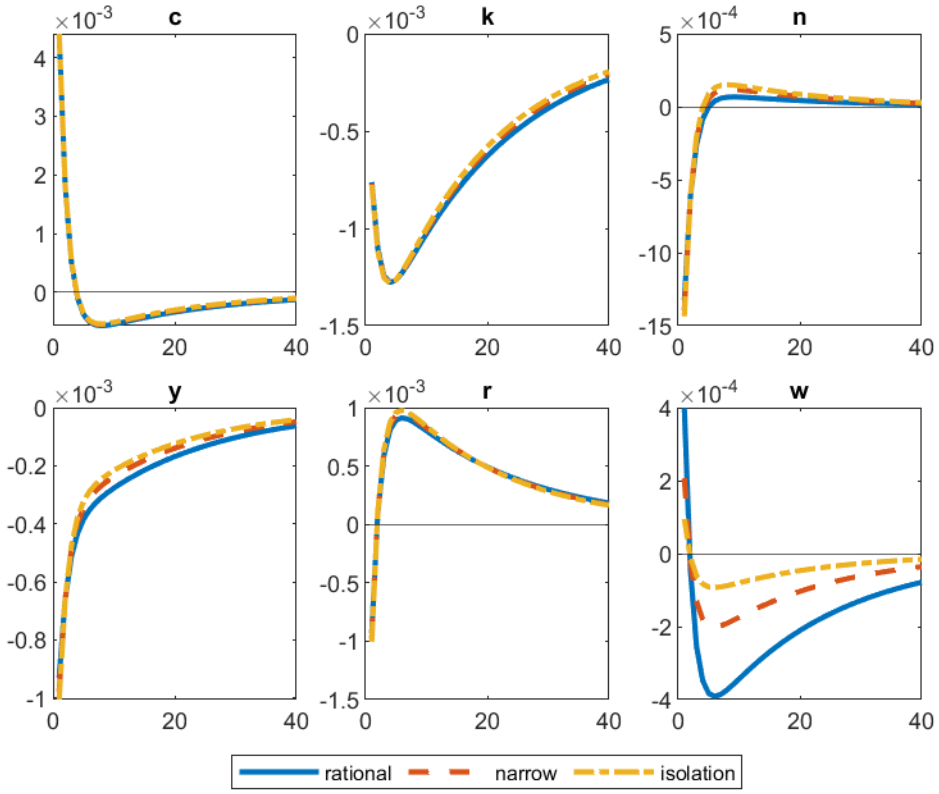


Note: Dotted lines illustrate the shifts of supply and demand curves for a rational firm. Solid lines illustrate the shifts of supply and demand curves for a narrow-framing firm with $\lambda_w < 1$.

The Effect of Noisy Rental Rate Signal and Noisy Demand Signal Figure 11 presents the IRFs to an investment shock for firms with narrow framing on the rental rate only. As in the case of a productivity shock, narrow framing on rental rate does not significantly change the pattern of impulse responses from the rational benchmark. It hardly changes the dynamic behaviors of endogenous variables, except for dampening the wage response. The intuition is similar to our previous discussion in the case of a productivity shock: the dampened shifts of capital demand curve are mostly undone by the amplified response of labor, which in turn mutes the influence of narrow framing on consumption and capital. Figure 12 illustrates the IRFs to an investment shock for firms that only receive a noisy signal on firm demand. Unlike the case of a productivity shock, the noisy signal on firm demand has almost no effect on the response of the economy to an investment shock. The reason is that the noisy signal on firm demand affects the economy primarily by dampening the direct effect of shocks that hit the firm on impact. In the case of a demand-side shock, like an investment shock or discount factor shock, since the shock hits the household directly instead of the firms, the attenuating direct effect of noisy demand signal is muted.

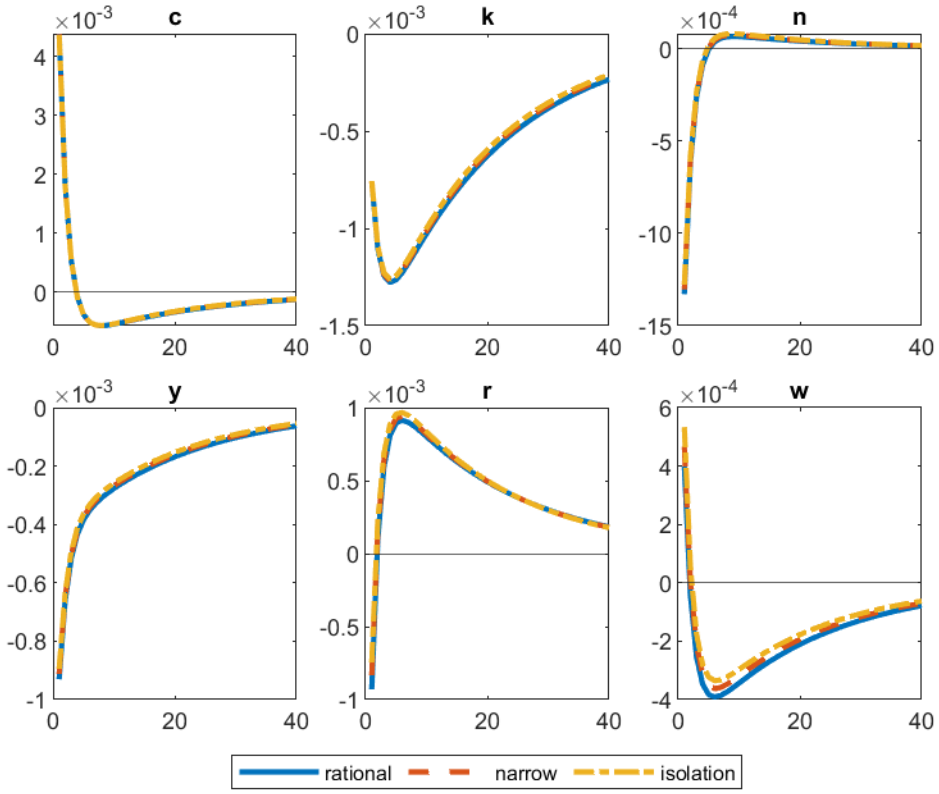
Summary In sum, compared with the case of a productivity shock, the IRFs of endogenous variables to an investment shock under narrow framing are not significantly different from the IRFs produced in a standard RBC model. The labor manager's narrow framing on rental

Figure 11: IRFs: Investment Shock, Noisy Rental Rate Signal Only



Note: λ_d and λ_w are set to 1. The "rational" line corresponds to the case where $\lambda_r = 1$. The "narrow" line corresponds to the case where $\lambda_r = 0.5$. The "isolation" line corresponds to the case where $\lambda_r = 0$.

Figure 12: IRFs: Investment Shock, Noisy Demand Signal Only



Note: λ_w and λ_r are set to 1. The "rational" line corresponds to the case where $\lambda_d = 1$. The "narrow" line corresponds to the case where $\lambda_d = 0.5$. The "isolation" line corresponds to the case where $\lambda_d = 0$.

rate of capital and the firm's uncertainty about demand can only produce tiny deviations from the rational benchmark. The capital manager's narrow framing on wage rate is again the friction that generates the largest aggregate deviations from the rational benchmark.

3.2.3 Additional Results

In Appendix A.4, I illustrate the IRFs of rational and narrow-framing firms to a one-standard-deviation shock to the discount factor. The conclusions are almost identical to my previous discussions about the case of investment shock, since in the RBC model, the impact and propagation mechanisms of investment and discount factor shocks are very similar to each other. Essentially, they can be interpreted as shocks to Euler equation: they change the effective real interest rate, and hence change the consumption and capital accumulation of the household. The only difference between these two shocks is that, besides the disturbance on Euler equation, the investment shock also introduces a small disturbance to the resource constraint. As we discussed above, this slightly lowers current consumption and capital accumulation. Therefore, capital decreases slightly more in response to an investment shock than a discount factor shock.

One may wonder what will happen if I introduce the three information frictions simultaneously as I did in the partial equilibrium analysis. As discussed above, in the case of a productivity shock, noisy signals on wage and rental rate amplify the business cycle while the noisy signal on firm demand dampens the business cycle. Therefore, if we have the amplifying frictions and dampening frictions simultaneously in the model, the behaviors of IRFs can be complicated. This makes it challenging to identify information frictions from IRFs.

4 Conclusion

This paper proposes narrow framing as a deviation from the standard assumption of perfect information sharing and coordination within firms, and studies its implications for firms' behavior in both partial equilibrium and general equilibrium. I let the capital and labor managers operate on different, non-nested information sets and assume that each manager knows only their own price perfectly but receives noisy signals about the other manager's price as well as the demand factor. With the information friction, managers cannot reason

through the ramifications of shocks on the other manager's behavior, which dampens their indirect, strategic responses. I show that in partial equilibrium, narrow framing attenuates the response of capital and labor choices to shocks in input prices and demand factors. Relating it to the misoptimization and misallocation literature, I show that narrow framing can be a potential cause of state-dependent misoptimization at the firm level and misallocation of resources in aggregate. In the general equilibrium analysis, I embed narrow-framing firms into an otherwise standard real business cycle (RBC) model and study its properties. I find that narrow framing can generate sizable aggregate impacts on the IRFs of endogenous variables to productivity and investment shocks. Future work will investigate empirically the plausibility of narrow framing at the firm level and quantify the aggregate effect in general equilibrium.

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A Appendix

A.1 Log-linearizing Decision Rules

I illustrate how I get the log-linearized first-order conditions (16) and (17) from the Bayesian Nash equilibrium (BNE) conditions (14) and (15). The left-hand side of (14) can be written as

$$
\begin{aligned}
&E\left[D^{1/\epsilon}F(K(s^k),N(s^n))^{-1/\epsilon}F_k(K(s^k),N(s^n))\middle|s^k\right] \\
&=\bar{D}^{1/\epsilon}E\left[\exp\left(\frac{1}{\epsilon}d\right)\exp\left(-\frac{1}{\epsilon}\log F(\bar{K}\exp(k),\bar{N}\exp(n))\right)\exp\left(\log F_k(\bar{K}\exp(k),\bar{N}\exp(n))\right)\middle|s^k\right]
\end{aligned}
\tag{97}
$$

Knowing that

$$
\begin{aligned}
&\exp\left(-\frac{1}{\epsilon}\log F(\bar{K}\exp(k),\bar{N}\exp(n))\right) \\
&\approx\exp\left\{-\frac{1}{\epsilon}\left[\log F(\bar{K},\bar{N})+\frac{F_k(\bar{K},\bar{N})}{F(\bar{K},\bar{N})}\bar{K}(\exp(k)-1)+\frac{F_n(\bar{K},\bar{N})}{F(\bar{K},\bar{N})}\bar{N}(\exp(n)-1)\right]\right\} \\
&\approx F(\bar{K},\bar{N})^{-1/\epsilon}\exp\left\{-\frac{1}{\epsilon}(\gamma_k k+\gamma_n n)\right\}
\end{aligned}
$$

and

$$
\begin{aligned}
&\exp\left(\log F_k(\bar{K}\exp(k),\bar{N}\exp(n))\right) \\
&\approx\exp\left\{F_k(\bar{K},\bar{N})+\frac{F_{kk}(\bar{K},\bar{N})}{F_k(\bar{K},\bar{N})}\bar{K}(\exp(k)-1)++\frac{F_{kn}(\bar{K},\bar{N})}{F_k(\bar{K},\bar{N})}\bar{N}(\exp(n)-1)\right\} \\
&\approx F_k(\bar{K},\bar{N})\exp\left\{\gamma_{kk}k+\gamma_{kn}n\right\}
\end{aligned}
$$

Plugging them into (97), we get an approximation of the left-hand side of (14)

$$
\bar{D}^{1/\epsilon}F(\bar{K},\bar{N})^{-1/\epsilon}F_k(\bar{K},\bar{N})E\left[\exp\left(\frac{1}{\epsilon}d-\frac{1}{\epsilon}(\gamma_k k+\gamma_n n)+\gamma_{kk}k+\gamma_{kn}n\right)\middle|s^k\right]
$$

and the right-hand side of (14) can be written as

$$
\frac{\epsilon}{\epsilon-1}\bar{R}\exp(r)
$$

Hence

$$\underbrace{\bar{D}^{1/\epsilon}F(\bar{K},\bar{N})^{-1/\epsilon}F_k(\bar{K},\bar{N})}_{A}\,E\left[\exp\left(\frac{1}{\epsilon}d-\frac{1}{\epsilon}(\gamma_k k+\gamma_n n)+\gamma_{kk}k+\gamma_{kn}n\right)\middle|s^k\right]=\underbrace{\frac{\epsilon}{\epsilon-1}}_{B}\bar{R}\exp(r) \tag{98}$$

By our definition of $\bar{K}$ and $\bar{N}$ (i.e., the choice of capital and labor for a rational firm when input prices and demand are set to their average values $\bar{R},\bar{D},\bar{W}$), A and B cancel out, and therefore we get (16). Reapplying the same procedure to Equation (15) yields (17).

A.2 Solving Log-linearized Decision Rules

I illustrate how to solve decision rules from Equations (16) and (17). Guess that the decision rule takes the form

$$k = \alpha_r r+\alpha_w\hat{w}+\alpha_d\hat{d} \tag{99}$$
$$n = \beta_r\hat{r}+\beta_w w+\beta_d\hat{d} \tag{100}$$

Plug them into the log-linearized first-order conditions (16) and (17):

$$\begin{aligned}\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)(\alpha_r r+\alpha_w\tilde{w}+\alpha_d\tilde{d})+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)E\left[\beta_r\tilde{r}+\beta_w w+\beta_d\tilde{d}|r,\tilde{w},\tilde{d}\right]+\frac{1}{\epsilon}E[d|r,\tilde{w},\tilde{d}] &= r\\ \left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)E[\alpha_r r+\alpha_w\tilde{w}+\alpha_d\tilde{d}|\tilde{r},w,\tilde{d}]+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\left[\beta_r\tilde{r}+\beta_w w+\beta_d\tilde{d}\right]+\frac{1}{\epsilon}E[d|\tilde{r},w,\tilde{d}] &= w\end{aligned}$$

Since we assumed that all signals are Gaussian and that all the priors are Gaussian with zero mean, we have

$$E[\tilde{r}|r]=r, E[\tilde{w}|w]=w$$

and

$$E[r|\tilde{r}]=\lambda_r\tilde{r}, E[w|\tilde{w}]=\lambda_w\tilde{w}, E[d|\tilde{d}]=\lambda_d\tilde{d}$$

where

$$\lambda_r\equiv\frac{\sigma_{\epsilon,r}^{-2}}{\sigma_{\epsilon,r}^{-2}+\sigma_r^{-2}}, \lambda_w\equiv\frac{\sigma_{\epsilon,w}^{-2}}{\sigma_{\epsilon,w}^{-2}+\sigma_w^{-2}}, \lambda_d\equiv\frac{\sigma_{\epsilon,d}^{-2}}{\sigma_{\epsilon,d}^{-2}+\sigma_d^{-2}}$$

are the weights that the managers put on the noisy signals that they receive. We get

$$\left[\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_r+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\beta_r\right]r+\left[\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_w+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\lambda_w\beta_w\right]\tilde{w} + \left[\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_d+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\beta_d+\frac{1}{\epsilon}\lambda_d\right]\tilde{d}=r \tag{101}$$

and

$$\left[\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\lambda_r\alpha_r+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\beta_r\right]\tilde{r}+\left[\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_w+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\beta_w\right]w + \left[\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_d+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\beta_d+\frac{1}{\epsilon}\lambda_d\right]\tilde{d}=w \tag{102}$$

If the guessed decision rules (99) and (100) are indeed consistent with Bayesian Nash equilibrium, Equations (101) and (102) should hold for any r, w, d and all realizations of signals $\tilde{r}, \tilde{w}, \tilde{d}$. Therefore, the coefficients on both sides of (101), (102) must coincide, which defines a system of six linear equations for six unknown coefficients α_i, β_i for $i = r, w, d$: for α_r and β_r, we have

$$\begin{cases}\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_r+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\beta_r & =1\\ \left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\lambda_r\alpha_r+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\beta_r & =0\end{cases} \tag{103}$$

For α_w and β_w, we have

$$\begin{cases}\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_w+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\lambda_w\beta_w & =0\\ \left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\lambda_w\alpha_w+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\beta_w & =1\end{cases} \tag{104}$$

and finally, for α_d and β_d, we have

$$\begin{cases}\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_d+\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\beta_d+\frac{1}{\epsilon}\lambda_d & =0\\ \left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\alpha_d+\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\beta_d+\frac{1}{\epsilon}\lambda_d & =0\end{cases} \tag{105}$$

Applying Cramer's rule to (103), (104) and (105), we can easily compute these coefficients:

$$\alpha_r(\lambda_r) = \frac{\gamma_{nn} - \frac{1}{\epsilon}\gamma_n}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_r\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)} \tag{106}$$

$$\beta_r(\lambda_r) = \frac{-\lambda_r\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_r\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)} \tag{107}$$

$$\alpha_w(\lambda_w) = \frac{-\lambda_w\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_w\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)} \tag{108}$$

$$\beta_w(\lambda_w) = \frac{\gamma_{kk} - \frac{1}{\epsilon}\gamma_k}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_w\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)} \tag{109}$$

$$\alpha_d(\lambda_d) = \frac{\frac{1}{\epsilon}(\gamma_{kn} - \gamma_{nn})\lambda_d}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)} \tag{110}$$

$$\beta_d(\lambda_d) = \frac{\frac{1}{\epsilon}(\gamma_{nk} - \gamma_{kk})\lambda_d}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)} \tag{111}$$

and this completes the calculation of the Bayesian Nash equilibrium decision rules.

A.3 Proofs in the main text

First, we observe that, with our assumptions on the production function (4), we have

$$\gamma_{kk}\gamma_{nn} - \gamma_{nk}\gamma_{kn} = \frac{F_{kk}K}{F_k}\frac{F_{nn}N}{F_n} - \frac{F_{kn}N}{F_k}\frac{F_{nk}K}{F_n} = \frac{KN}{F_kF_n}(F_{kk}F_{nn} - F_{kn}F_{nk}) > 0 \tag{112}$$

where the last inequality comes from the assumption that F is increasing in (K, N) and that F is concave. As a result,

$$\gamma_{kk}\gamma_{nn} - \lambda_i\gamma_{nk}\gamma_{kn} \geq \gamma_{kk}\gamma_{nn} - \gamma_{nk}\gamma_{kn} > 0 \tag{113}$$

for any $\lambda_i \in [0, 1]$.

Therefore, the denominators in the coefficients

$$\begin{aligned}
&\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_i\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) \\
&= \underbrace{(\gamma_{kk}\gamma_{nn} - \lambda_i\gamma_{kn}\gamma_{nk})}_{>0} - \underbrace{\frac{1}{\epsilon}\gamma_k(\gamma_{nn} - \lambda_i\gamma_{kn})}_{<0} - \underbrace{\frac{1}{\epsilon}(\gamma_{kk} - \lambda_i\gamma_{nk})}_{<0} + \underbrace{\frac{1}{\epsilon^2}(1 - \lambda_i)\gamma_n\gamma_k}_{>0} > 0
\end{aligned} \tag{114}$$

are positive for any $\lambda_i \in [0,1]$, $i = r, w, d$. Whether the coefficients are positive or negative depends on the signs of numerators. Again, by assumption (4), we have

$$\gamma_{nn} < 0, \gamma_{kk} < 0, \gamma_k > 0, \gamma_n > 0, \gamma_{kn} > 0, \gamma_{nk} > 0$$

therefore we know for sure that

$$\alpha_r(\lambda_r) < 0, \beta_w(\lambda_w) < 0, \alpha_d(\lambda_d) > 0, \beta_d(\lambda_d) > 0$$

Now I present the proofs of the propositions and corollaries in the main text.

Proof of Proposition 1

Proof. As I mentioned in the main text, the case of rational firms is nested by setting $\lambda_r = \lambda_w = 1$. To compare cross-price elasticities of a narrow-framing firm and a rational firm, we simply compare the values of $\alpha_w(\lambda_w)$ and $\alpha_w(1)$, as well as the values of $\beta_r(\lambda_r)$ and $\beta_r(1)$.

If $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k > 0$, $\beta_r(\lambda_r) < 0$, and

$$\text{sgn}\left(\frac{d\beta_r(\lambda_r)}{d\lambda_r}\right) = \text{sgn}\left(\frac{d}{d\lambda_r}\frac{-1}{1/\lambda_r + \text{const.}}\right) < 0$$

hence $\beta_r(\lambda_r) > \beta_r(1)$ for $\lambda_r \in [0,1)$, i.e.,

$$\frac{\partial n^{rational}}{\partial r} < \frac{\partial n^{narrow}}{\partial r} < 0$$

If, on the other hand, $\gamma_{nk} - \frac{1}{\epsilon}\gamma_k < 0$, then $\beta_r(\lambda_r) > 0$, and

$$\text{sgn}\left(\frac{d\beta_r(\lambda_r)}{d\lambda_r}\right) = \text{sgn}\left(\frac{d}{d\lambda_r}\frac{1}{1/\lambda_r + \text{const.}}\right) > 0$$

hence $\beta_r(\lambda_r) < \beta_r(1)$ for $\lambda_r \in [0,1)$, i.e.,

$$0 < \frac{\partial n^{narrow}}{\partial r} < \frac{\partial n^{rational}}{\partial r}$$

Applying the same argument (except changing the condition to whether $\gamma_{kn} - \frac{1}{\epsilon}\gamma_n > 0$) to $\alpha_w(\lambda_w)$ yields the same result.

Hence, no matter which direction the cross-price elasticity takes, narrow framing always dampens the response of labor to rental rate shocks and the response of capital to wage shocks. □

Proof of Proposition 2

Proof. As shown above, $\alpha_r(\lambda_r) < 0$. Also, note that

$$\operatorname{sgn}\left(\frac{d\alpha_r(\lambda_r)}{d\lambda_r}\right) = \operatorname{sgn}\left(\frac{d}{d\lambda_r}\frac{-1}{\text{const.} - \lambda_r\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)}\right)$$

Clearly,

- If $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) \geq 0$, or equivalently,

  $$\epsilon \in \left[\max\left\{\frac{\gamma_n}{\gamma_{kn}}, \frac{\gamma_k}{\gamma_{nk}}\right\}, \infty\right) \cup \left(1, \min\left\{\frac{\gamma_n}{\gamma_{kn}}, \frac{\gamma_k}{\gamma_{nk}}\right\}\right],$$

  then

  $$\operatorname{sgn}\left(\frac{d}{d\lambda_r}\frac{-1}{\text{const.} - \lambda_r\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)}\right) \leq 0$$

  so that $\alpha_r(\lambda_r) > \alpha_r(1)$ for $\lambda_r \in [0, 1)$, i.e.,

  $$\frac{\partial k^{rational}}{\partial r} < \frac{\partial k^{narrow}}{\partial r} < 0$$

  In this case, narrow framing dampens the own-price elasticity.

- If $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right) < 0$, or equivalently,

  $$\epsilon \in \left(\min\left\{\frac{\gamma_n}{\gamma_{kn}}, \frac{\gamma_k}{\gamma_{nk}}\right\}, \max\left\{\frac{\gamma_n}{\gamma_{kn}}, \frac{\gamma_k}{\gamma_{nk}}\right\}\right)$$

  then

  $$\operatorname{sgn}\left(\frac{d}{d\lambda_r}\frac{-1}{\text{const.} - \lambda_r\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)}\right) > 0$$

so that $\alpha_r(\lambda_r) < \alpha_r(1)$ for $\lambda_r \in [0, 1)$, i.e.,

$$\frac{\partial k^{narrow}}{\partial r} < \frac{\partial k^{rational}}{\partial r} < 0$$

In this case, narrow framing amplifies the own-price elasticity.

With our assumption (4), however, we have

$$\frac{\gamma_k}{\gamma_{nk}} = \frac{F_k(\bar{K}, \bar{N})F_n(\bar{K}, \bar{N})}{F(\bar{K}, \bar{N})F_{nk}(\bar{K}, \bar{N})}$$

and

$$\frac{\gamma_n}{\gamma_{kn}} = \frac{F_k(\bar{K}, \bar{N})F_n(\bar{K}, \bar{N})}{F(\bar{K}, \bar{N})F_{kn}(\bar{K}, \bar{N})}$$

Since $F_{kn} = F_{nk}$, hence $\frac{\gamma_k}{\gamma_{nk}} = \frac{\gamma_n}{\gamma_{kn}}$, so that as long as $\frac{\gamma_k}{\gamma_{nk}} = \frac{\gamma_n}{\gamma_{kn}} \leq 1$, $\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)$ will always be positive, and the own-price elasticity is dampened. □

Proof of Proposition 3

Proof. From (110) and (111), we know that the demand elasticities of capital and labor are positive and increasing with λ_d, hence

$$0 < \alpha_d(\lambda_d) < \alpha_d(1)$$

and

$$0 < \beta_d(\lambda_d) < \beta_d(1)$$

for $\lambda_d \in [0, 1)$. It follows immediately that narrow framing dampens the capital and labor's response to demand shocks. □

Proof of Proposition 4

Proof. As stated in (34),

$$\begin{aligned}\frac{\partial E[y|r, w, d]}{\partial r} &= \gamma_k \alpha_r(\lambda_r) + \gamma_n \beta_r(\lambda_r) \\ &= \frac{\gamma_k\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_r\gamma_n\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)}{\left(\gamma_{kk} - \frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn} - \frac{1}{\epsilon}\gamma_n\right) - \lambda_r\left(\gamma_{kn} - \frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk} - \frac{1}{\epsilon}\gamma_k\right)}\end{aligned}$$

First of all, the numerator

$$\gamma_k\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)-\lambda_r\gamma_n\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)=\gamma_k\gamma_{nn}-\frac{1}{\epsilon}\gamma_k\gamma_n(1-\lambda_r)-\lambda_r\gamma_n\gamma_{nk}<0$$

so that $\frac{\partial E[y|r,w,d]}{\partial r}<0$. Next,

$$\begin{aligned}
&\operatorname{sgn}\left(\frac{d}{d\lambda_r}\frac{\partial E[y|r,w,d]}{\partial r}\right)\\
=&\operatorname{sgn}\left\{-\gamma_n\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\left[\left(\gamma_{kk}-\frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)-\lambda_r\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\right]\right.\\
&\left.-(-1)\left(\gamma_{kn}-\frac{1}{\epsilon}\gamma_n\right)\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\left(\gamma_k\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)-\lambda_r\gamma_n\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\right)\right\}\\
=&\operatorname{sgn}\left\{\left(\gamma_{nk}-\frac{1}{\epsilon}\gamma_k\right)\left(\gamma_{nn}-\frac{1}{\epsilon}\gamma_n\right)\left(\gamma_k\gamma_{kn}-\gamma_n\gamma_{kk}\right)\right\}
\end{aligned}$$

The sign of this depends on $\gamma_{nk}-\frac{1}{\epsilon}\gamma_k$:

- If $\gamma_{nk}-\frac{1}{\epsilon}\gamma_k>0$, then

  $$0>\frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial r}>\frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial r}$$

  in this case the response of output to rental rate shocks is dampened;

- If $\gamma_{nk}-\frac{1}{\epsilon}\gamma_k<0$, then

  $$0>\frac{\partial E\left[y^{rational}|r,w,d\right]}{\partial r}>\frac{\partial E\left[y^{narrow}|r,w,d\right]}{\partial r}$$

  in this case the response of output to rental rate shocks is amplified.

The arguments for the response of output to wage rate shocks follow the same lines as above.

From (36), the response of output to a demand shock is positive and will always be dampened since both capital and labor choice respond to a demand shock in a positive, dampened manner. □

Proof of Proposition 5

Proof. Since

$$p = \frac{1}{\epsilon}(d - y)$$

we have

$$\frac{\partial E[p|r,w,d]}{\partial r} = -\frac{1}{\epsilon}\frac{\partial E[y|r,w,d]}{\partial r}$$
$$\frac{\partial E[p|r,w,d]}{\partial w} = -\frac{1}{\epsilon}\frac{\partial E[y|r,w,d]}{\partial w}$$

and

$$\frac{\partial E[p|r,w,d]}{\partial d} = \frac{1}{\epsilon}\left(1 - \frac{\partial E[y|r,w,d]}{\partial d}\right)$$

and the results follow immediately from Proposition 4. □

Proof of Corollary 1

Proof. The result follows directly from Propositions 1–4 and the fact that

$$\frac{\gamma_k}{\gamma_{nk}} = \frac{\gamma_n}{\gamma_{kn}} = 1 < \epsilon$$

To double check the result, I replace $F(K,N) = K^\alpha N^{1-\alpha}$ in (106) to (111):

$$\alpha_r(\lambda_r) = \frac{-\alpha - \frac{1}{\epsilon}(1-\alpha)}{\left(\alpha - 1 - \frac{1}{\epsilon}\alpha\right)\left(-\alpha - \frac{1}{\epsilon}(1-\alpha)\right) - \lambda_r\left(\alpha - \frac{1}{\epsilon}\alpha\right)\left(1 - \alpha - \frac{1}{\epsilon}(1-\alpha)\right)} \quad (115)$$

$$\beta_r(\lambda_r) = \frac{-\left(\alpha - \frac{1}{\epsilon}\alpha\right)\lambda_r}{\left(\alpha - 1 - \frac{1}{\epsilon}\alpha\right)\left(-\alpha - \frac{1}{\epsilon}(1-\alpha)\right) - \lambda_r\left(\alpha - \frac{1}{\epsilon}\alpha\right)\left(1 - \alpha - \frac{1}{\epsilon}(1-\alpha)\right)} \quad (116)$$

$$\alpha_w(\lambda_w) = \frac{-\lambda_w\left(1 - \alpha - \frac{1}{\epsilon}(1-\alpha)\right)}{-\lambda_w\left(1 - \alpha - \frac{1}{\epsilon}(1-\alpha)\right)\left(\alpha - \frac{1}{\epsilon}\alpha\right) + \left(\alpha - 1 - \frac{1}{\epsilon}\alpha\right)\left(-\alpha - \frac{1}{\epsilon}(1-\alpha)\right)} \quad (117)$$

$$\beta_w(\lambda_w) = \frac{\alpha - 1 - \frac{1}{\epsilon}\alpha}{-\lambda_w\left(1 - \alpha - \frac{1}{\epsilon}(1-\alpha)\right)\left(\alpha - \frac{1}{\epsilon}\alpha\right) + \left(\alpha - 1 - \frac{1}{\epsilon}\alpha\right)\left(-\alpha - \frac{1}{\epsilon}(1-\alpha)\right)} \quad (118)$$

$$\alpha_d(\lambda_d) = \lambda_d \quad (119)$$

$$\beta_d(\lambda_d) = \lambda_d \quad (120)$$

and the results presented in the main text are indeed correct. □

Proof of Proposition 6

Proof. Part 1 follows directly from the main text by applying the definition of the misopti-

mization wedge.

Part 2 follows from Corollary 1:

$$\alpha_r(\lambda_r) > \alpha_r(1), \alpha_w(\lambda_w) > \alpha_w(1), \alpha_d(\lambda_d) = \lambda_d < 1 = \alpha_d(1)$$

Hence the average misoptimization is increasing with r_t, w_t and decreasing with d_t.

Part 3 requires us to analyze the monotonicity of $\alpha_w(\lambda_w)^2\sigma^2_{\epsilon,w}, \beta_r(\lambda_r)^2\sigma^2_{\epsilon,r}$ and $\lambda_d^2\sigma^2_{\epsilon,d}$, with respect to the variance of idiosyncratic noises $\sigma^2_{\epsilon,w}, \sigma^2_{\epsilon,r}, \sigma^2_{\epsilon,d}$.

First, look into $\lambda_d^2\sigma^2_{\epsilon,d}$. We have

$$\begin{aligned}
\lambda_d^2\sigma^2_{\epsilon,d} &= \frac{\sigma_d^4}{(\sigma_d^2 + \sigma^2_{\epsilon,d})^2}\sigma^2_{\epsilon,d} \\
&= \frac{\sigma^4}{\left(\frac{\sigma_d^2}{\sigma_{\epsilon,d}} + \sigma_{\epsilon,d}\right)^2} \\
&= \frac{\sigma_d^4}{\frac{\sigma_d^4}{\sigma^2_{\epsilon,d}} + 2\sigma_d^2 + \sigma^2_{\epsilon,d}}
\end{aligned} \tag{121}$$

The denominator of (121) is decreasing with $\sigma^2_{\epsilon,d}$ (or equivalently, increasing with λ_d) when $\sigma^2_{\epsilon,d} \leq \sigma_d^2$ (which is equivalent to $\lambda_d \geq 1/2$) and increasing with $\sigma^2_{\epsilon,d}$ (or equivalently, decreasing with λ_d) when $\sigma^2_{\epsilon,d} > \sigma_d^2$ (which is equivalent to $\lambda_d < 1/2$). Hence $\lambda_d^2\sigma^2_{\epsilon,d}$ is decreasing when $\lambda_d \geq 1/2$ and increasing when $\lambda_d < 1/2$.

Next, consider $\alpha_w(\lambda_w)^2\sigma^2_{\epsilon,w}$. We have

$$\begin{aligned}
\operatorname{sgn}\left\{\frac{\partial \alpha_w(\lambda_w)^2\sigma^2_{\epsilon,w}}{\partial \sigma^2_{\epsilon,w}}\right\} &= \operatorname{sgn}\left\{2\alpha_w(\lambda_w)\frac{\partial \alpha_w}{\partial \lambda_w}\frac{\partial \lambda_w}{\partial \sigma^2_{\epsilon,w}}\sigma^2_{\epsilon,w} + \alpha_w(\lambda_w)^2\right\} \\
&= -\operatorname{sgn}\left\{2\frac{\partial \alpha_w}{\partial \lambda_w}\frac{\partial \lambda_w}{\partial \sigma^2_{\epsilon,w}}\sigma^2_{\epsilon,w} + \alpha_w(\lambda_w)\right\}
\end{aligned}$$

and note that

$$
\begin{aligned}
&2\frac{\partial\alpha_w}{\partial\lambda_w}\frac{\partial\lambda_w}{\partial\sigma^2_{\epsilon,w}}\sigma^2_{\epsilon,w} \\
&= 2\frac{-(1-\alpha)(1-1/\epsilon)\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)+\alpha(1-\alpha)(1-1/\epsilon)^2(-1)(1-\alpha)(1-1/\epsilon)\lambda_w}{\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)^2} \\
&\times \frac{-\sigma^2_w}{(\sigma^2_w+\sigma^2_{\epsilon,w})^2}\sigma^2_{\epsilon,w} \\
&= 2\frac{-(1-\alpha)(1-1/\epsilon)(\alpha(1-\alpha)(1-1/\epsilon)^2+1/\epsilon)}{\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)^2}\frac{-\sigma^2_w}{\sigma^2_w+\sigma^2_{\epsilon,w}}\frac{\sigma^2_{\epsilon,w}}{\sigma^2_w+\sigma^2_{\epsilon,w}} \\
&= 2\frac{(1-\alpha)(1-1/\epsilon)(\alpha(1-\alpha)(1-1/\epsilon)^2+1/\epsilon)}{\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)^2}\frac{\sigma^2_w}{\sigma^2_w+\sigma^2_{\epsilon,w}}\frac{\sigma^2_{\epsilon,w}}{\sigma^2_w+\sigma^2_{\epsilon,w}} \\
&= 2\frac{(1-\alpha)(1-1/\epsilon)(\alpha(1-\alpha)(1-1/\epsilon)^2+1/\epsilon)}{\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)^2}\lambda_w(1-\lambda_w)
\end{aligned}
$$

Hence

$$
\begin{aligned}
\operatorname{sgn}\left\{2\frac{\partial\alpha_w}{\partial\lambda_w}\frac{\partial\lambda_w}{\partial\sigma^2_{\epsilon,w}}\sigma^2_{\epsilon,w}+\alpha_w(\lambda_w)\right\} &= \operatorname{sgn}\left\{2\frac{(1-\alpha)(1-1/\epsilon)(\alpha(1-\alpha)(1-1/\epsilon)^2+1/\epsilon)}{\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)^2}\lambda_w(1-\lambda_w)+\alpha_w\right\} \\
&= \operatorname{sgn}\Bigg\{2(1-\alpha)(1-1/\epsilon)(\alpha(1-\alpha)(1-1/\epsilon)^2+1/\epsilon)\lambda_w(1-\lambda_w) \\
&\quad -\lambda_w(1-\alpha)(1-1/\epsilon)\left((1-\lambda_w)(1-\alpha)\alpha(1-1/\epsilon)^2+1/\epsilon\right)\Bigg\} \\
&= \operatorname{sgn}\Big\{(1-\alpha)(1-1/\epsilon)^3\alpha(1-\alpha)\lambda_w(1-\lambda_w)+(1-2\lambda_w)(1-\alpha)(1-1/\epsilon \\
&= \operatorname{sgn}\left\{\alpha(1-\alpha)(1-1/\epsilon)^2(1-\lambda_w)+(1-2\lambda_w)\frac{1}{\epsilon}\right\} \\
&= \operatorname{sgn}\left\{\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{1}{\epsilon}-\left[\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{2}{\epsilon}\right]\lambda_w\right\}
\end{aligned}
$$

and therefore

$$
\operatorname{sgn}\left\{\frac{\partial\alpha_w(\lambda_w)^2\sigma^2_{\epsilon,w}}{\partial\sigma^2_{\epsilon,w}}\right\} = \operatorname{sgn}\left\{-\left[\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{1}{\epsilon}\right]+\left[\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{2}{\epsilon}\right]\lambda_w\right\} \tag{122}
$$

Therefore, $\alpha_w\sigma^2_{\epsilon,w}$ is increasing with $\sigma^2_{\epsilon,w}$ (or equivalently, decreasing with λ_w) if and only if $-\left[\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{1}{\epsilon}\right]+\left[\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{2}{\epsilon}\right]\lambda_w \geq 0$, or equivalently,

$$
\lambda_w \geq \frac{\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{1}{\epsilon}}{\alpha(1-\alpha)(1-1/\epsilon)^2+\frac{2}{\epsilon}} \equiv \Lambda \tag{123}
$$

$\alpha_w \sigma^2_{\epsilon,w}$ is decreasing with $\sigma^2_{\epsilon,w}$ (or equivalently, increasing with λ_w) if and only if $\lambda_w < \Lambda$.

The argument for $\beta_r \sigma^2_{\epsilon,r}$ is the same. □

Proof of Proposition 7

Proof. It is easy to see that

$$\begin{aligned}
\alpha_r(\lambda_r) - \beta_r(\lambda_r) - 1 &= \frac{-\alpha(1-1/\epsilon) - 1/\epsilon + \alpha(1-1/\epsilon)\lambda_r}{(1-\lambda_r)\alpha(1-\alpha)(1-1/\epsilon)^2 + 1/\epsilon} + 1 \\
&= \frac{-\alpha(1-\lambda_r)(1-1/\epsilon) - 1/\epsilon + (1-\lambda_r)\alpha(1-\alpha)(1-1/\epsilon)^2 + 1/\epsilon}{(1-\lambda_r)\alpha(1-\alpha)(1-1/\epsilon)^2 + 1/\epsilon} \\
&= \frac{(1-\lambda_r)\alpha(1-1/\epsilon)((1-\alpha)(1-1/\epsilon) - 1)}{(1-\lambda_r)\alpha(1-\alpha)(1-1/\epsilon)^2 + 1/\epsilon} < 0
\end{aligned}$$

Hence the capital-labor wedge decreases with r_t. Similarly,

$$\alpha_w(\lambda_w) - \beta_w(\lambda_w) - 1 < 0$$

Hence the capital-labor wedge decreases with w_t.

The results about dispersion of capital-labor wedge follow directly from the proof of Part 3 of Proposition 6. □

Proof of Proposition 8

Proof. I first transform the log-linearized decision rules (18) and (19) to the original K_{it} and N_{it}:

$$K_{it} = \bar{K} \exp\{\alpha_r(\lambda_r) r_t + \alpha_w(\lambda_w)(w_t + \epsilon^w_{it}) + \alpha_d(\lambda_d)(y_t + (\epsilon - 1)a_{it} + \epsilon^d_{it})\} \quad (124)$$

$$N_{it} = \bar{N} \exp\{\beta_r(\lambda_r)(r_t + \epsilon^r_{it}) + \beta_w(\lambda_w) w_t + \beta_d(\lambda_d)(y_t + (\epsilon - 1)a_{it} + \epsilon^d_{it})\} \quad (125)$$

Then, aggregate K_{it} and N_{it}, and compute the individual firm's share of aggregate inputs:

$$\frac{K_{it}}{K_t} = \frac{A_{it}^{\lambda_d(\epsilon-1)} \exp\{\alpha_w(\lambda_w)\epsilon^w_{it} + \lambda_d \epsilon^d_{it}\}}{E\left[A_{it}^{\lambda_d(\epsilon-1)} \exp\{\alpha_w(\lambda_w)\epsilon^w_{it} + \lambda_d \epsilon^d_{it}\}\right]} = \frac{A_{it}^{\lambda_d(\epsilon-1)} \exp\{\alpha_w(\lambda_w)\epsilon^w_{it} + \lambda_d \epsilon^d_{it}\}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right] E\left[\exp\{\alpha_w(\lambda_w)\epsilon^w_{it} + \lambda_d \epsilon^d_{it}\}\right]} \quad (126)$$

$$\frac{N_{it}}{N_t} = \frac{A_{it}^{\lambda_d(\epsilon-1)} \exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}}{E\left[A_{it}^{\lambda_d(\epsilon-1)} \exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}\right]} = \frac{A_{it}^{\lambda_d(\epsilon-1)} \exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right] E\left[\exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}\right]} \quad (127)$$

where I plugged in $\alpha_d(\lambda_d) = \beta_d(\lambda_d) = \lambda_d$ from (119) and (120), and used the fact that A_{it} are independent of the idiosyncratic noises.

Therefore,

$$\begin{aligned} A_{it}\frac{K_{it}^\alpha N_{it}^{1-\alpha}}{K_t^\alpha N_t^{1-\alpha}} &= A_{it}\left[\frac{A_{it}^{\lambda_d(\epsilon-1)} \exp\{\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d\}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right] E\left[\exp\{\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d\}\right]}\right]^\alpha \left[\frac{A_{it}^{\lambda_d(\epsilon-1)} \exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right] E\left[\exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}\right]}\right]^1 \\ &= A_{it}\frac{A_{it}^{\lambda_d(\epsilon-1)}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right]} \frac{\exp\{\alpha(\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d) + (1-\alpha)\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}}{E\left[\exp\{\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d\}\right]^\alpha E\left[\exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}\right]^{1-\alpha}} \end{aligned}$$

and therefore

$$\begin{aligned} A_t &= \frac{\left[\int_0^1 (A_{it}K_{it}^\alpha N_{it}^{1-\alpha})^{1-1/\epsilon}\, di\right]^{\epsilon/(\epsilon-1)}}{K_t^\alpha N_t^{1-\alpha}} \\ &= \frac{E\left[A_{it}^{(1+\lambda_d(\epsilon-1))(1-1/\epsilon)}\right]^{\frac{\epsilon}{\epsilon-1}} E\left[\exp\left\{(1-1/\epsilon)(\alpha(\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d) + (1-\alpha)\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d)\right\}\right]^{\frac{\epsilon}{\epsilon-1}}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right] E\left[\exp\{\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d\}\right]^\alpha E\left[\exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}\right]^{1-\alpha}} \end{aligned}$$

Define

$$H \equiv \frac{E\left[\exp\left\{(1-1/\epsilon)(\alpha(\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d) + (1-\alpha)\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d)\right\}\right]^{\frac{\epsilon}{\epsilon-1}}}{E\left[\exp\{\alpha_w(\lambda_w)\epsilon_{it}^w + \lambda_d\epsilon_{it}^d\}\right]^\alpha E\left[\exp\{\beta_r(\lambda_r)\epsilon_{it}^r + \lambda_d\epsilon_{it}^d\}\right]^{1-\alpha}}$$

It is known that for a log-normal random variable X with $\log X \sim N(\mu, \sigma^2)$, then

$$E[X] = \exp\left\{\mu + \frac{1}{2}\sigma^2\right\} \quad (128)$$

Using this property of log-normal distribution, we have

$$
\begin{aligned}
H &= \frac{\exp\left\{\frac{\epsilon-1}{\epsilon}\frac{1}{2}\left[\alpha^2\alpha_w^2\sigma_{\epsilon,w}^2+(1-\alpha)^2\beta_r^2\sigma_{\epsilon,r}^2+\lambda_d^2\sigma_{\epsilon,d}^2\right]\right\}}{\exp\left\{\frac{1}{2}\alpha_w^2\alpha\sigma_{\epsilon,w}^2+\frac{1}{2}\beta_r^2\sigma_{\epsilon,r}^2(1-\alpha)+\frac{1}{2}\lambda_d^2\sigma_{\epsilon,d}^2\right\}} \\
&= \exp\left\{\underbrace{\frac{1}{2}((1-1/\epsilon)\alpha^2-\alpha)\alpha_w^2\sigma_{\epsilon,w}^2}_{<0}+\underbrace{\frac{1}{2}((1-\alpha)^2(1-1/\epsilon)-(1-\alpha))\beta_r^2\sigma_{\epsilon,r}^2}_{<0}-\frac{1}{2\epsilon}\lambda_d^2\sigma_{\epsilon,d}^2\right\} \\
&< 1
\end{aligned} \tag{129}
$$

Note that with the result of Part 3, Proposition 6, we know that $\alpha_w^2\sigma_{\epsilon,w}^2$ is decreasing with λ_w if and only if $\lambda_w \geq \Lambda$. Therefore, H is increasing with λ_w if and only if $\lambda_w \geq \Lambda$, and $\lambda_r \geq \Lambda$.

Next, we deal with the term with A_{it}:

$$
\begin{aligned}
\frac{E\left\{A_{it}^{(1+\lambda_d(\epsilon-1))(1-1/\epsilon)}\right]^{\frac{\epsilon}{\epsilon-1}}}{E\left[A_{it}^{\lambda_d(\epsilon-1)}\right]} &= \frac{\exp\left\{\frac{\epsilon-1}{\epsilon}\frac{1}{2}(1+\lambda_d(\epsilon-1))^2\sigma_a^2\right\}}{\exp\left\{\frac{1}{2}\lambda_d^2(\epsilon-1)^2\sigma_a^2\right\}} \\
&= \exp\left\{\frac{1}{2}\left(\epsilon-1-\frac{(\epsilon-1)^2}{\epsilon}(1-\lambda_d)^2\right)\sigma_a^2\right\} \\
&= \frac{\exp\left\{\frac{1}{2}(\epsilon-1)\sigma_a^2\right\}}{\exp\left\{\frac{1}{2}\left(\frac{(\epsilon-1)^2}{\epsilon}(1-\lambda_d)^2\right)\sigma_a^2\right\}} \\
&= E\left[A_{it}^{\epsilon-1}\right]^{\frac{1}{\epsilon-1}}\underbrace{\exp\left\{-\frac{1}{2}\left(\frac{(\epsilon-1)^2}{\epsilon}(1-\lambda_d)^2\right)\sigma_a^2\right\}}_{<1 \text{ and decreasing with } \sigma_a^2}
\end{aligned} \tag{130}
$$

Lastly, we simply let

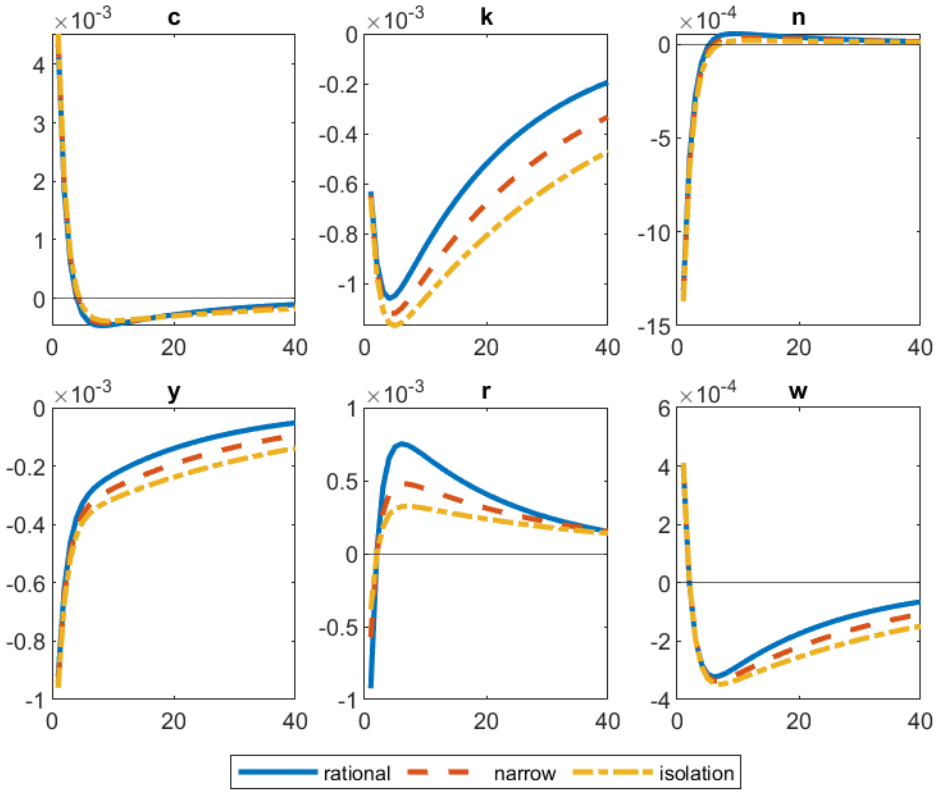
$$
H(\lambda_r,\lambda_w,\lambda_d,\sigma_{\epsilon,r}^2,\sigma_{\epsilon,w}^2,\sigma_{\epsilon,d}^2,\sigma_a^2) \equiv \exp\left\{-\frac{1}{2}\left(\frac{(\epsilon-1)^2}{\epsilon}(1-\lambda_d)^2\right)\sigma_a^2\right\} \times H < 1 \tag{131}
$$

which completes the proof. □

A.4 Narrow Framing and IRFs to Discount Factor Shocks

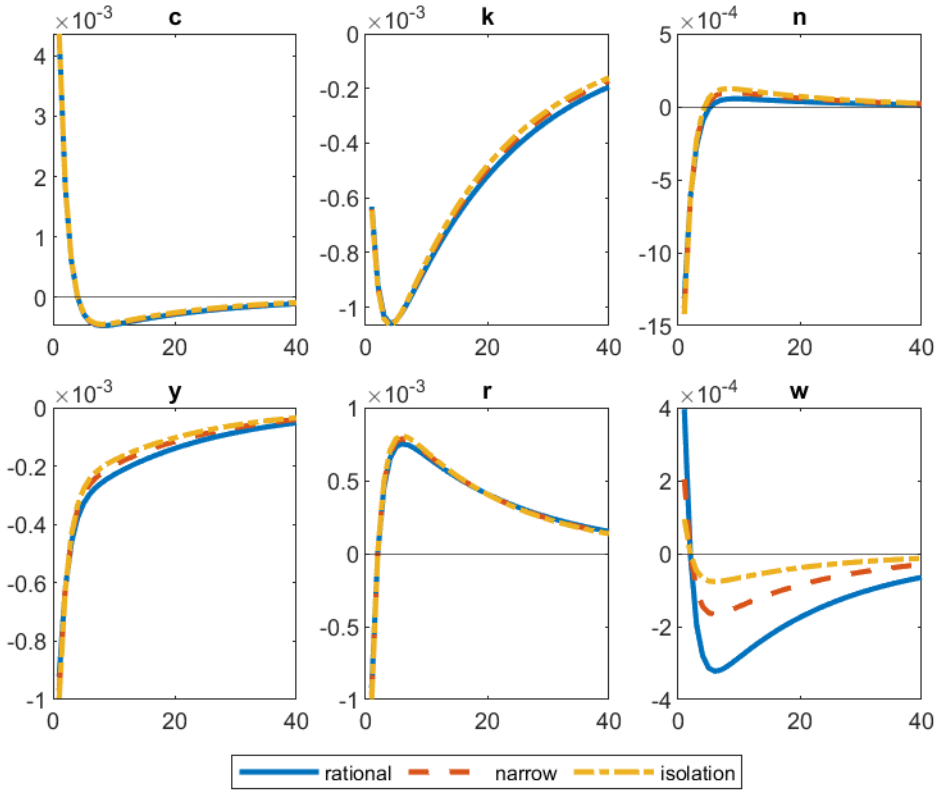
I attach IRFs to a discount factor shock for firms with different degrees of narrow framing.

Figure 13: IRFs: Discount Factor Shock, Noisy Wage Signal Only



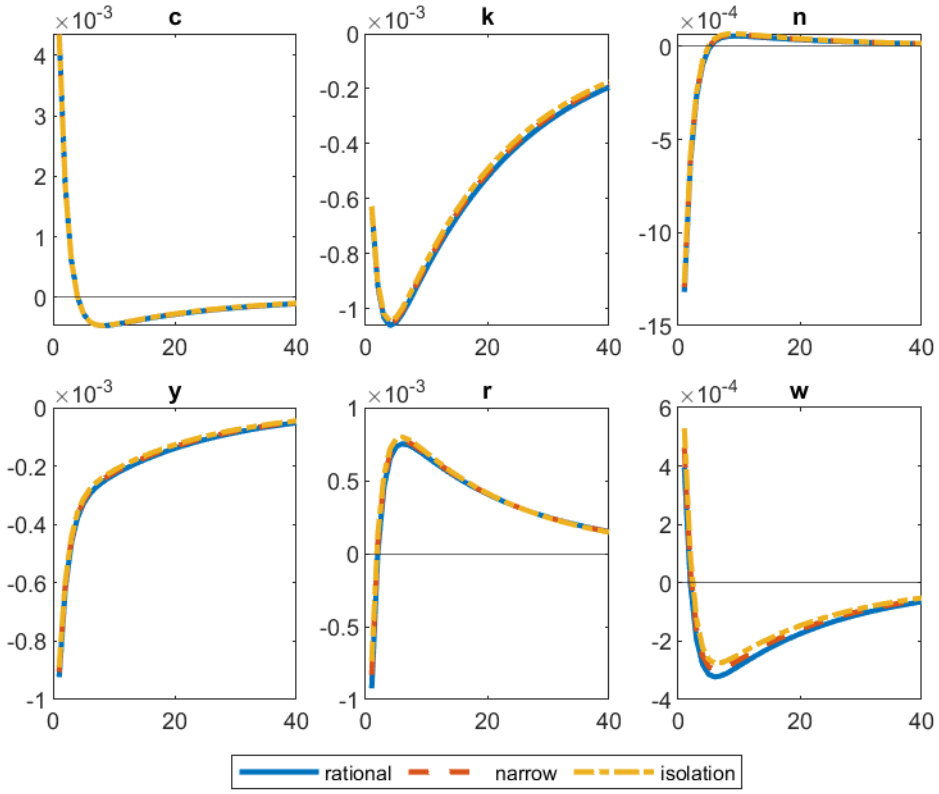
Note: λ_d and λ_r are set to 1. The "rational" line corresponds to the case where $\lambda_w = 1$. The "narrow" line corresponds to the case where $\lambda_w = 0.5$. The "isolation" line corresponds to the case where $\lambda_w = 0$.

Figure 14: IRFs: Discount Factor Shock, Noisy Rental Rate Signal Only



Note: λ_d and λ_w are set to 1. The "rational" line corresponds to the case where $\lambda_r = 1$. The "narrow" line corresponds to the case where $\lambda_r = 0.5$. The "isolation" line corresponds to the case where $\lambda_r = 0$.

Figure 15: IRFs: Discount Factor Shock, Noisy Demand Signal Only



Note: λ_r and λ_w are set to 1. The "rational" line corresponds to the case where $\lambda_d = 1$. The "narrow" line corresponds to the case where $\lambda_d = 0.5$. The "isolation" line corresponds to the case where $\lambda_d = 0$.