

TA Session 3: Firm Static Optimization

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- Firm's Optimization Problem with Proportional Taxes;
- Firm's Optimization Problem with Lump-sum Taxes & Fixed Operational Costs;
- Policy exercise: employment commitment program in Germany.

A useful graph

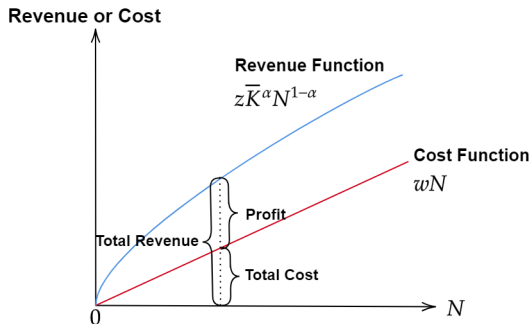


Figure: Revenue & Cost Function

Firm's Problem with a Proportional Producer Tax

- The firm pays t units of consumption goods to the government for each unit of output it produces. Then, the firm's problem is:

$$\max_N \pi = (1 - t)z\bar{K}^\alpha(N)^{1-\alpha} - wN + (1 - d)\bar{K}$$

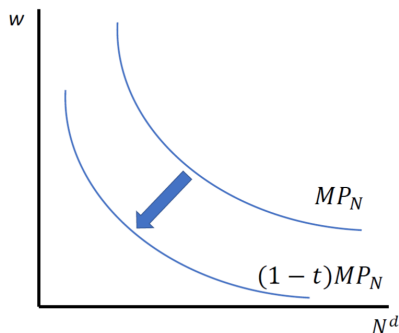
- The first order condition is

$$(1 - t)(1 - \alpha)z\bar{K}^\alpha(N)^{-\alpha} = w$$

(after-tax marginal product of labor equals real wage)

- End up with optimal labor demand $N^d(w, t)$.

A proportional tax reduces labor demand

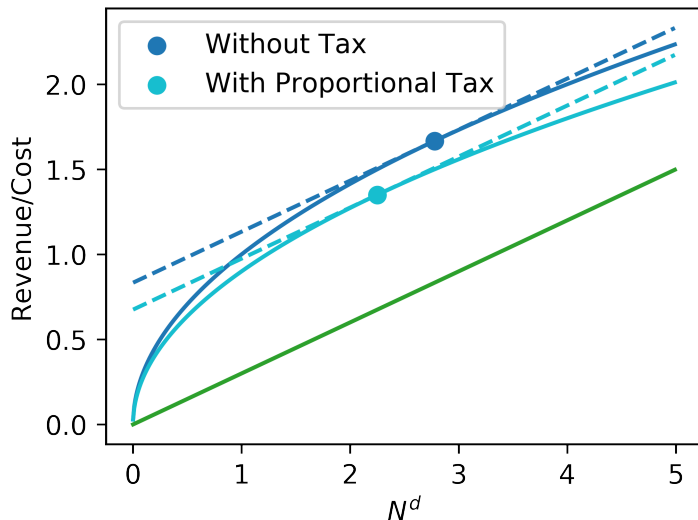


- Labor demand is given by

$$w = (1 - t)(1 - \alpha)z \left(\frac{\bar{K}}{N^d} \right)^\alpha$$

- Intuition: raising a tax rate decreases profits from producing one additional unit of goods, and the firm reduces labor input given the same level of real wage.

A proportional tax reduces labor demand



An increase in proportional tax reduces labor demand

Labor demand is given by

$$w = (1 - t)(1 - \alpha)z \left(\frac{\bar{K}}{N^d} \right)^\alpha$$

Hence

$$N^d = \left(\frac{(1 - t)(1 - \alpha)z \bar{K}^\alpha}{w} \right)^{1/\alpha}$$

- How does an increase in t changes N^d ?
- Calculate the partial derivative of N^d with respect to t :

$$\frac{\partial N^d}{\partial t} = - \left(\frac{(1 - \alpha)z \bar{K}^\alpha}{w} \right)^{1/\alpha} (1 - t)^{1/\alpha - 1} \frac{1}{\alpha} < 0$$

- Hence, an increase in proportional tax reduces labor demand.

Firm's Problem with Lump-sum Tax

- The firm pays a lump sum T units of consumption goods to the government. Then, the firm's problem is:

$$\max_N \pi = z\bar{K}^\alpha (N)^{1-\alpha} - wN + (1-d)P\bar{K} - T$$

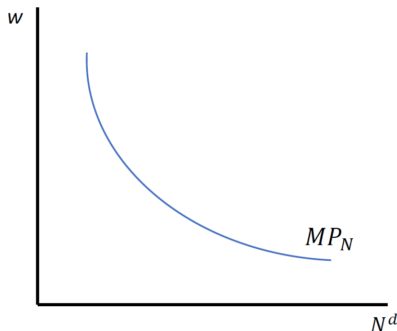
- The first order condition is

$$(1-\alpha)z\bar{K}^\alpha (N^d)^{-\alpha} = w$$

(marginal product of labor equals real wage)

- Lump-sum Tax doesn't appear in first order condition.

A Lump Sum Tax Does Not Change Labor Demand

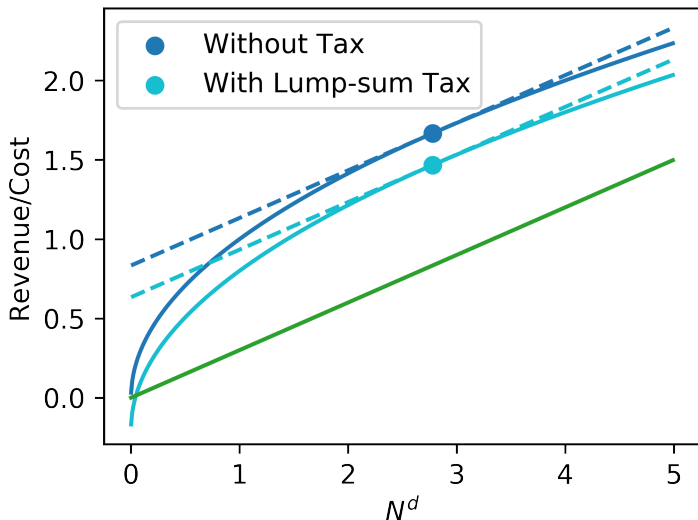


- Labor demand is given by

$$w = (1 - \alpha)z \left(\frac{\bar{K}}{N^d} \right)^\alpha$$

- Intuition: lump-sum tax doesn't change the marginal benefit or the marginal cost of hiring a labor.

A Lump Sum Tax Does Not Change Labor Demand



Firm's Problem with Fixed Operational Cost

- Variable Costs vs. Fixed Costs:
 - Variable costs: costs that scale up with production;
 - e.g. capital, labor, energy, etc.;
 - Fixed costs: costs independent of scale of production;
 - e.g. rent of land, getting license, loan repayment, posting recruitment info, etc.
- Total cost is the sum of variable cost and fixed cost:

$$\mathcal{C}(N) = wN + F$$

where F denotes fixed cost.

Firm's Problem with Fixed Operational Cost

- With fixed cost, profit function is given by (here, assume $d = 1$, capital fully depreciates)

$$z\bar{K}N^{1-\alpha} - wN - F$$

- Firm chooses N to maximize this profit function.
- First order condition:

$$z\bar{K}N^{-\alpha}(1 - \alpha) = w$$

again, same as the no-tax benchmark.

- Fixed cost is mathematically same as a lump-sum tax.

Firm's Problem with Fixed Operational Cost

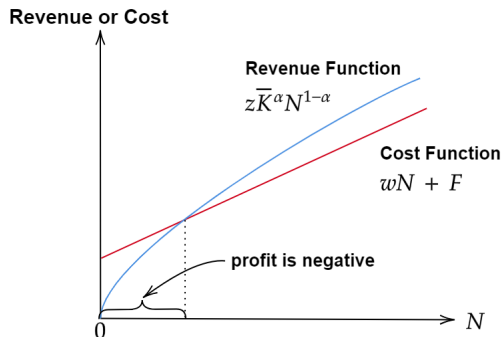


Figure: Revenue and Cost Function with Fixed Cost

Firm's Problem with Fixed Operational Cost

- If firms cannot earn positive profit even when they are optimizing, they would exit.
- Without fixed cost, firms never exit (easiest to see from graph);
- Lump-sum taxes and Fixed costs may lead firms to exit, if T or F is high enough.
- To see that algebraically, recall that optimal labor input is

$$N^d = \left(\frac{(1 - \alpha)z\bar{K}^\alpha}{w} \right)^{1/\alpha}$$

so that the maximum profit that a firm could earn is

$$\pi^* = z\bar{K}(N^d)^{1-\alpha} - wN^d - F$$

The firm would exit if $\pi^* < 0$.

Firm's Problem with Fixed Operational Cost

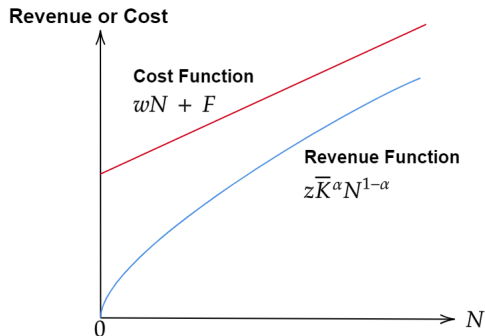


Figure: Firms may exit if the fixed cost is too high.

Firm's Problem with Employment Commitment

- Besides taxes, government can regulate labor market in many other ways;
- In many former communist economies,
 - Transitioned to market economy in 1990s;
 - New capital investment, obsolete labor skills, massive unemployment;
 - Government response: labor market regulations;
- After 1991, privatized East German firms were registered in an employment commitment program:
 - Government sets a target level of employment for each firm;
 - If the firm's hiring falls short of the target, it is punished by the government.
- We are interested in the economic consequence of this “employment commitment” policy.

Firm's Problem with Employment Commitment

- From now on, assume capital stock $\bar{K} = 1$, so that production function becomes

$$Y = zN^{1-\alpha}$$

- Suppose the government sets an employment target $\bar{N}$;
- If the firm hires less than $\bar{N}$:
 - for each missing amount of employment, the government charges a punishment $w\gamma$;

Hence the cost of hiring is given by:

- 1 If $N < \bar{N}$,

$$\mathcal{C}(N) = wN + \gamma w(\bar{N} - N) = \gamma w\bar{N} + (1 - \gamma)wN$$

- 2 If $N \geq \bar{N}$,

$$\mathcal{C}(N) = wN$$

Firm's Problem with Employment Commitment

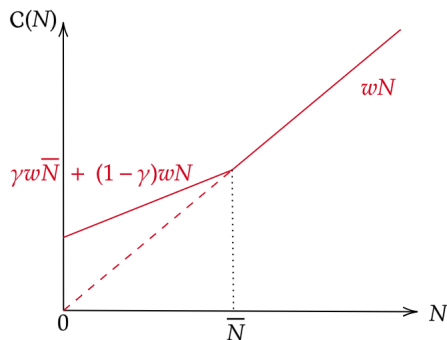


Figure: Cost Function under Employment Commitment

Firm's Problem with Employment Commitment

- Employment commitment program is akin to providing a proportional subsidy while charging a lump-sum tax on firms with low employment:
 - The punishment makes the cost function flatter on the left of $\bar{N}$ while introducing a “fixed cost”;
 - this fixed cost may lead some firms to exit ($\downarrow N$);
 - meanwhile, marginal cost of hiring is smaller in the punishment zone ($N \leq \bar{N}$);
 - hence, the punishment encourages hires below $\bar{N}$ ($\uparrow N$).

Firm's Problem with Employment Commitment

- Now the firm's problem becomes

$$\max_N zN^{1-\alpha} - \mathcal{C}(N)$$

where

$$\mathcal{C}(N) = \begin{cases} \gamma w \bar{N} + (1 - \gamma)wN & \text{if } N < \bar{N} \\ wN & \text{if } N \geq \bar{N} \end{cases}$$

- Suppose w and $\bar{N}$ are the same for all firms;
- We are interested in how this employment commitment program affect firms with different productivity level z .

Graphical Illustration

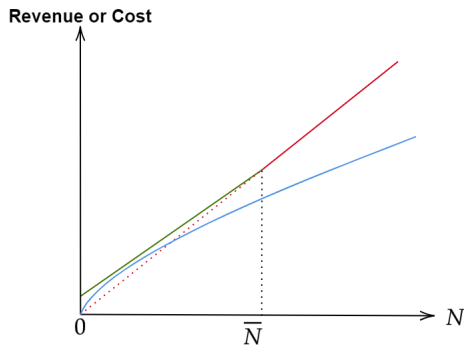


Figure: Super-low productivity firms exit, since they cannot afford the punishment.

Graphical Illustration

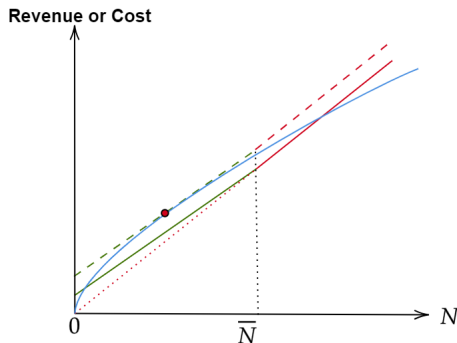


Figure: Low productivity firms are distorted at “subsidized optimum” level of employment.

Graphical Illustration

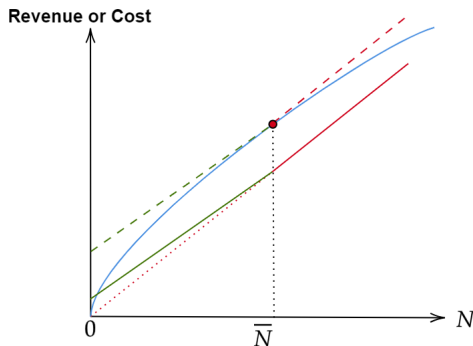


Figure: Median productivity firms are distorted and “bunched” at the committed employment level $\bar{N}$.

Graphical Illustration

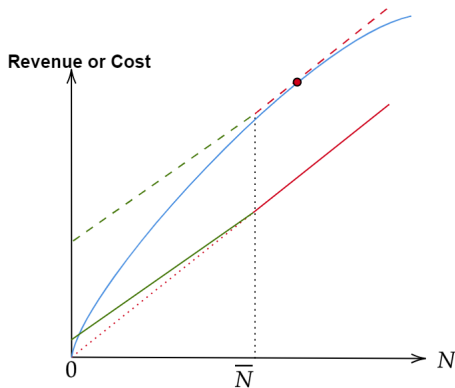


Figure: High productivity firms are not distorted by employment commitment.

Distribution of Employment across Firms at Different Productivity Levels

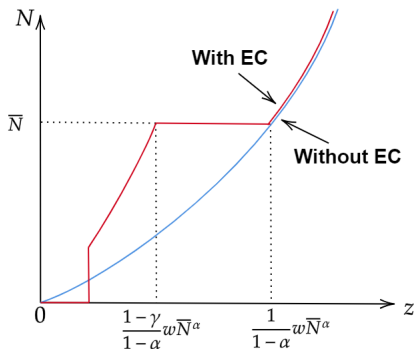


Figure: Distribution of employment across firms at different productivity levels, with Employment Commitment (red) vs. without (blue)

Takeaway

- Super-low-productivity firms quit, do not produce and hire labor;
- It's mainly low- and median-productivity firms that increase their hiring;
- High-productivity firms are not affected;
- Strikingly, a bunch of median-productivity firms hire exactly the committed amount $\bar{N}$;
 - On the left of $\bar{N}$: their marginal product is always greater than the marginal cost under punishment $(1 - \gamma)w \implies$ increase production;
 - On the right of $\bar{N}$: their marginal product is always smaller than the marginal cost $w \implies$ decrease production;
 - Hence, it's optimal for them to hire exactly at $\bar{N}$.
- Low productivity firm would rather hire below $\bar{N}$ and pay the punishment, since they cannot afford the high marginal cost beyond $\bar{N}$.

Takeaway

- Increase hiring is not necessarily good for firms, since low- and median- productivity firms are earning less profit compared to the unregulated case.
- Whether employment commitment program increases employment depends on the share of exiting firms vs. the share of low & median productivity firms in the economy.
- If we have good firm-level data, we can test these hypotheses: check out the working paper “Committing to Grow: Privatizations and Firm Dynamics in East Germany”, by Akcigit, Alp, Diegmann and Serrano-Velarde.

Algebraic Solution (Optional)

- How to solve firm's problem with employment commitment?
- Discuss case by case.
- ① If $N \geq \bar{N}$, the problem becomes

$$\max_{N \geq \bar{N}} zN^{1-\alpha} - wN$$

First order condition:

$$z(1-\alpha)N^{-\alpha} - w = 0 \implies N^* = \left(\frac{z}{w}\right)^{1/\alpha} (1-\alpha)^{1/\alpha}$$

Need to check whether $N^* \geq \bar{N}$:

- $N^* \geq \bar{N}$ is equivalent to $z \geq \frac{w\bar{N}^\alpha}{(1-\alpha)}$;
- if $z \geq \frac{w\bar{N}^\alpha}{(1-\alpha)}$, the labor input choice is not distorted;

Algebraic Solution (Optional)

- How to solve firm's problem with employment commitment?
- Discuss case by case.
- ① If $N \geq \bar{N}$, the problem becomes

$$\max_{N \geq \bar{N}} zN^{1-\alpha} - wN$$

First order condition:

$$z(1 - \alpha)N^{-\alpha} - w = 0 \implies N^* = \left(\frac{z}{w}\right)^{1/\alpha} (1 - \alpha)^{1/\alpha}$$

Need to check whether $N^* \geq \bar{N}$:

- If $z < \frac{w\bar{N}^\alpha}{(1-\alpha)}$, then $N^* < \bar{N}$, firm's optimal hiring falls below $\bar{N}$;
- Hiring at N^* will be subject to the punishment!
- Hence, for these firms, hiring $\bar{N}$ gives them max. profit.

Algebraic Solution (Optional)

- ② If $N < \bar{N}$, the problem becomes

$$\max_{N < \bar{N}} zN^{1-\alpha} - (1-\gamma)wN - \gamma w\bar{N}$$

First order condition:

$$z(1-\alpha)N^{-\alpha} - (1-\gamma)w = 0 \implies N^{**} = \left(\frac{z}{w}\right)^{1/\alpha} \left(\frac{1-\alpha}{1-\gamma}\right)^{1/\alpha}$$

Again, need to check whether $N^{**} < \bar{N}$:

- $N^{**} < \bar{N}$ is equivalent to $z < \frac{1-\gamma}{1-\alpha} w \bar{N}^\alpha$;
- if $z < \frac{1-\gamma}{1-\alpha} w \bar{N}^\alpha$, the firm will hire at N^{**} (“subsidized optimum”);
- It is a distorted level of employment that is higher than what it would have chosen without employment commitment;

Algebraic Solution (Optional)

- ② If $N < \bar{N}$, the problem becomes

$$\max_{N < \bar{N}} zN^{1-\alpha} - (1-\gamma)wN - \gamma w\bar{N}$$

First order condition:

$$z(1-\alpha)N^{-\alpha} - (1-\gamma)w = 0 \implies N^{**} = \left(\frac{z}{w}\right)^{1/\alpha} \left(\frac{1-\alpha}{1-\gamma}\right)^{1/\alpha}$$

Again, need to check whether $N^{**} < \bar{N}$:

- if $z \geq \frac{1-\gamma}{1-\alpha} w \bar{N}^\alpha$, then $N^{**} \geq \bar{N}$;
i.e. it's tempting for the subsidized firm to hire beyond $\bar{N}$;
- However, hiring N^{**} has a higher unit cost (w instead of $(1-\gamma)w$)!
- Hence, for such firms, it's best to hire at $\bar{N}$.

Algebraic Solution (Optional)

Let's summarize what we found:

- ① If $z \geq \frac{w\bar{N}^\alpha}{(1-\alpha)}$, then

$$N^d = N^* = \left(\frac{z}{w}\right)^{1/\alpha} (1-\alpha)^{1/\alpha}$$

- ② If $z < \frac{1-\gamma}{1-\alpha} w\bar{N}^\alpha$, then

$$N^d = N^{**} = \left(\frac{z}{w}\right)^{1/\alpha} \left(\frac{1-\alpha}{1-\gamma}\right)^{1/\alpha}$$

- ③ If $\frac{1-\gamma}{1-\alpha} w\bar{N}^\alpha \leq z < \frac{w\bar{N}^\alpha}{(1-\alpha)}$, then

$$N^d = \bar{N}$$