

TA Session 11: Growth Accounting

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- Take the production function from Solow model:

$$Y = zF(K, N)$$

- We want to measure how much of the growth in aggregate output (Y) over a given period of time is accounted for by growth in
 - the inputs to production (K, N), and
 - total factor productivity (z).
- This exercise is called **Growth Accounting**.

Growth Accounting with Cobb-Douglas Production Function

- We assume that

$$F(K, N) = K^{\alpha} N^{1-\alpha}$$

- $\alpha \in (0, 1)$ is the parameter that measures capital share of income, i.e. the proportion of national income that compensates the capital input.
- $1 - \alpha$ captures the labor share of income, i.e. the national income that compensates the labor input.
- Estimated from post-war US data: $\alpha \approx 0.3$.

Growth Accounting with Cobb-Douglas Production Function

- With measures of aggregate output, the capital input, and the labor input, denoted Y , K and N respectively, then total factor productivity z can be measured as a residual:

$$z \approx \frac{Y}{K^{0.3}N^{0.7}}$$

- Or, equivalently, take log on both sides:

$$\log z \approx \log Y - 0.3 \log K - 0.7 \log N$$

- We call z or $\log z$ “**Solow residual**”. It is an approximation of total factor productivity.

Interpreting Solow residual

$$\log z_{t+1} \approx \log Y_{t+1} - 0.3 \log K_{t+1} - 0.7 \log N_{t+1}$$

$$\log \hat{z}_t \approx \log Y_t - 0.3 \log K_t - 0.7 \log N_t$$

$$\Rightarrow \log \left(\frac{z_{t+1}}{z_t} \right) \approx \log \left(\frac{Y_{t+1}}{Y_t} \right) - 0.3 \log \left(\frac{K_{t+1}}{K_t} \right) - 0.7 \log \left(\frac{N_{t+1}}{N_t} \right)$$

- Recall Taylor approximation: for a small x ,

$$\log(1+x) \approx x \text{ and } \log(1-x) \approx -x$$

- Hence

$$\log \left(\frac{X_{t+1}}{X_t} \right) = \log \left(1 + \frac{\Delta X_{t+1}}{X_t} \right) \approx \frac{\Delta X_{t+1}}{X_t}$$

Interpreting Solow residual

- Apply the above trick to variable z_t , Y_t , K_t and N_t , we have

$$\Rightarrow \frac{\Delta z_{t+1}}{z_t} = \frac{\Delta Y_{t+1}}{Y_t} - 0.3 \frac{\Delta K_{t+1}}{K_t} - 0.7 \frac{\Delta N_{t+1}}{N_t}$$

- We can see that the growth of total factor productivity (represented by the percentage change of Solow residual) is the proportion of GDP growth that is **not** captured by the (weighted average of) growth in capital stock and labor input.

Growth accounting exercise: Post-war US data

Data preparation:

- Y : Real Gross Domestic Product, 1950-present
(FRED series GDPC1) [click here](#)
- K : Capital Stock at Constant National Prices, 1950-present
(FRED series RKNANPUSA666NRUG) [click here](#)
- N : Employment level, 1950-present
(FRED series CE160V) [click here](#)

We can thus calculate the time series of Solow residual by ¹

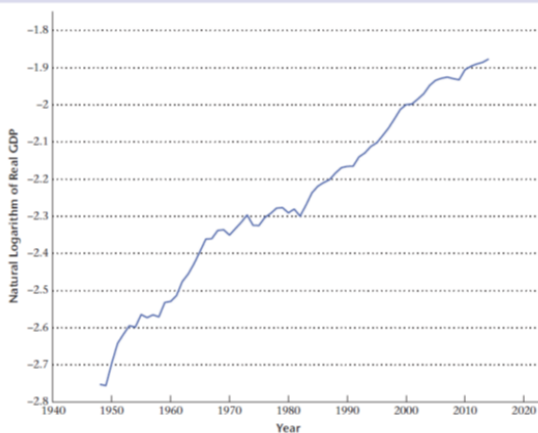
$$z \approx \frac{Y}{K^{0.3}N^{0.7}}$$

¹Since real GDP and capital stock are based on dollars in different years, you may also need inflation data to adjust for the discrepancy.

Growth accounting exercise: Post-war US data

Figure 7.23 Natural Log of the Solow Residual, 1948–2014

The Solow residual is a measure of total factor productivity.



Growth accounting exercise: Post-war US data

Table 7.2 Measured GDP, Capital Stock, Employment, and Solow Residual

Year	$\hat{Y}$ (billions of 2009 dollars)	$\hat{K}$ (billions of 2009 dollars)	$\hat{N}$ (millions)	$\hat{z}$
1950	2184.0	8004.2	58.9	8.50
1960	3108.7	11509.0	65.8	10.04
1970	4722.0	16819.7	78.7	12.00
1980	6450.4	22629.8	99.3	12.74
1990	8955.0	29286.6	118.8	14.44
2000	12682.2	37283.6	136.9	17.06
2010	14418.7	44944.9	139.9	18.24
2014	15961.7	47109.5	146.3	19.30

Table 7.3 Average Annual Growth Rates

Years	$\hat{Y}$	$\hat{K}$	$\hat{N}$	$\hat{z}$
1950–1960	3.6	3.7	1.1	1.7
1960–1970	4.3	3.9	1.8	1.8
1970–1980	3.2	3.0	2.4	0.6
1980–1990	3.3	2.6	1.8	1.3
1990–2000	3.5	2.4	1.4	1.7
2000–2009	1.4	2.1	0.2	0.7
2009–2014	2.1	0.9	0.9	1.1

- Solow residual is the first-step of anatomizing economic growth;
- On top of that, we can ask the following questions:
 - Are technological advances purely exogenous?
 - Is it possible for endogenous growth?
 - Many great economists contribute to this literature: Schumpeter, Romer, Akcigit, ...
- Future trends of productivity?
 - Robert Gordon argued that the productivity growth is stagnating and we should not expect economic growth to sustain in the future;
 - Joel Mokyr, Daron Acemoglu and a few others take a more optimistic perspective.
- People at UMD made great contribution to the measure of TFP: John Haltiwanger, Chad Syverson, ...