

TA Session 4: Market Clearing Conditions

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This week:

- Competitive equilibrium;
- Work out labor market clearing wage.

Next week: Review Session for Midterm 1.

- You may email me about the questions/topics that you want me to discuss in class (I will prioritize these questions);
- I will quickly go through course materials.

Competitive Equilibrium

- So far, we've discussed consumer and firm's optimization problem:
 - Consumers: given $\{w, \pi\}$, choose optimal consumption $c^*(w, \pi)$, leisure $l^*(w, \pi)$ and labor supply $N^s(w, \pi)$;
 - Firms: given $\{w\}$, choose optimal labor demand $N^d(w)$ and end up with maximum profit $\pi(w)$.
- Last step: market clearing, solve for equilibrium prices w^* and quantities $(c^*, l^*, (N^s)^*, (N^d)^*, \pi^*)$.
- There are two markets that need to be cleared:
 - 1 Goods market: $c^* = z\bar{K}^\alpha (N^*)^{1-\alpha}$;
This holds automatically if we plug the definition of profit into the consumer's budget constraint.
 - 2 Labor market: $N^s(w, \pi(w)) = N^d(w)$;
This solves equilibrium wage w^* and hence the equilibrium quantities (c^*, l^*, N^*, π^*) .
since $(N^s)^* = (N^d)^*$ in equilibrium, I simply use N^* to denote equilibrium employment.

Definition of labor market clearing

- Definition

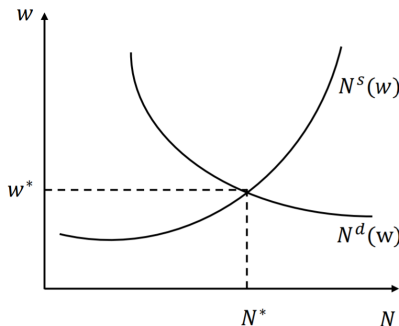
- Consumer and firm are each optimizing, given real wages
- Labor demand = labor supply

- Approach

- Find labor demand curve as a function of real wages, $N^d(w)$
- Find labor supply curve as a function of real wages, $N^s(w)$
- Find intersection

- Distinguish

- Exogenous variables:
 $z, K, \alpha, \gamma, h, t, T$
- Endogenous variables:
 w, N, Y, π



Derive $N^d(w)$ from the firm side

- State the firm's profit maximization problem (in real terms; assume capital fully depreciates):

$$\max_{N^d} \pi = z\bar{K}^\alpha (N^d)^{1-\alpha} - wN^d$$

- Take the FOC:

$$(1 - \alpha)z\bar{K}^\alpha (N^d)^{-\alpha} = w$$

- Solve for $N^d(w)$:

$$N^d(w) = \left(\frac{(1 - \alpha)z\bar{K}^\alpha}{w} \right)^{1/\alpha}$$

Derive $N^s(w)$ from the consumer side

- Consumer's utility maximization problem

$$\max_{c, l} \ln(c) + \gamma \ln(l) \text{ s.t. } c = w(h - l) + \pi$$

- Set up the Lagrangian:

$$L = \ln(c) + \gamma \ln(l) + \lambda[w(h - l) + \pi - c]$$

- Take the FOC:

$$\frac{1}{c} = \lambda \tag{1}$$

$$\frac{\gamma}{l} = \lambda w \tag{2}$$

$$c = w(h - l) + \pi \tag{3}$$

Derive $N^s(w)$ from the consumer side

- Combine equations (1) and (2) to eliminate λ :

$$I = \frac{\gamma c}{w}$$

- Plug the previous equation into equation (3) and solve:

$$c^*(w, \pi) = \frac{wh + \pi}{1 + \gamma}$$
$$I^*(w, \pi) = \frac{\gamma}{1 + \gamma} \frac{wh + \pi}{w}$$

- Solve for N^s as a function of w and π :

$$N^s(w, \pi) = h - I^*(w, \pi) = \frac{1}{1 + \gamma} \left(h - \frac{\gamma \pi}{w} \right)$$

- To close the market, labor supply and demand decisions will be only in terms of wages, so we must eliminate π .

Eliminate π I

- Taking profits from the firm's problem:

$$\pi = z\bar{K}^\alpha (N^d)^{1-\alpha} - wN^d$$

- Recall the optimal labor demand $N^d(w)$ is derived from firm's FOC:

$$N^d(w) = \left(\frac{(1-\alpha)z\bar{K}^\alpha}{w} \right)^{1/\alpha}$$

- Plug in $N^d(w)$ into profit function:

$$\pi(w) = z\bar{K}^\alpha \left(\frac{(1-\alpha)z\bar{K}^\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} - w \left(\frac{(1-\alpha)z\bar{K}^\alpha}{w} \right)^{\frac{1}{\alpha}}$$

- Simplifying:

$$\pi(w) = \alpha z\bar{K} \left(\frac{(1-\alpha)z}{w} \right)^{\frac{1-\alpha}{\alpha}}$$

Eliminate π II

- Profits are only a function of parameters, so labor supply is only a function of parameters and the real wage:

$$N^s(w, \pi(w)) = \frac{1}{1+\gamma} \left(h - \frac{\gamma \alpha z \bar{K} \left(\frac{(1-\alpha)z}{w} \right)^{\frac{1-\alpha}{\alpha}}}{w} \right) \quad (4)$$

$$= \frac{1}{1+\gamma} \left(h - \gamma \alpha z^{1/\alpha} \bar{K} (1-\alpha)^{\frac{1-\alpha}{\alpha}} w^{-1/\alpha} \right) \quad (5)$$

Equilibrium expressions for N^* and w^*

- Suppose $\bar{K} = 1$.
- To solve for the equilibrium wage w^* , use the labor market clearing condition $N^s(w, \pi(w)) = N^d(w)$:

$$\left(\frac{(1-\alpha)z}{w}\right)^{1/\alpha} = \frac{1}{1+\gamma} \left(h - \gamma\alpha \left(\frac{z(1-\alpha)^{1-\alpha}}{w} \right)^{\frac{1}{\alpha}} \right)$$

- Simplify:

$$\frac{(1-\alpha)z}{w^*} = \left(\frac{h(1-\alpha)}{(1+\gamma)(1-\alpha) + \alpha\gamma} \right)^\alpha$$

- Therefore:

$$w^* = (1-\alpha)z \left(\frac{(1+\gamma)(1-\alpha) + \alpha\gamma}{h(1-\alpha)} \right)^\alpha$$

- How does the productivity affect wages?

Equilibrium expressions for N^* and w^*

Plugging w^* in either N^d or N^s we get

$$N^* = \frac{(1 - \alpha)h}{1 + \gamma - \alpha}$$

- How does productivity (z) affect the labor? Why?
- What happens if the labor share changes?
- Play with the graph!