

TA Session 10: Solow Growth Model

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Roadmap

- Review of Solow Model;
- Golden Rule in Solow Model;
- Convergence in Solow Model.

Review: Solow Model I

- Capital accumulation equation in aggregate:

$$K' = (1 - d)K + s \underbrace{zF(K, N)}_{\equiv y}$$

where $F(K, N)$ is a production function and N is the size of labor force.

- Economic interpretation: the economy saves s fraction of output in capital for future productive use (s is exogenous).
- Assume that

$$F(K, N) = K^\alpha N^{1-\alpha}$$

so that per-capita production is equal to

$$\frac{F(K, N)}{N} = \frac{K^\alpha N^{1-\alpha}}{N} = \left(\frac{K}{N}\right)^\alpha \equiv k^\alpha$$

where $k \equiv \frac{K}{N}$ denotes per-capita capital stock.

Review: Solow Model II

- Divide both sides of capital accumulation equation by N , we get the law of motion for per-capita capital stock k :

$$\underbrace{\frac{K'}{N}}_{\equiv \frac{K'}{N'} \frac{N'}{N} = k'(1+n)} = (1-d) \underbrace{\frac{K}{N}}_{\equiv k} + sz \underbrace{\frac{F(K, N)}{N}}_{\equiv k^\alpha}$$

$$\implies (1+n)k' = (1-d)k + szk^\alpha$$

- We can compute the steady-state per-capita capital: set $k' = k = k^{ss}$, and we get

$$(1+n)k^{ss} = (1-d)k^{ss} + sz(k^{ss})^\alpha$$

$$\implies k^{ss} = \left(\frac{n+d}{sz} \right)^{\frac{1}{\alpha-1}}$$

Review: Solow Model III

- In this simple setup, there are two forces that drives per-capita capital stock down:
 - ① Labor force: since $k \equiv \frac{K}{N}$, more population $\implies \uparrow N, \downarrow k$;
 - ② Depreciation: higher d , higher depreciation, lower capital stock.
- On the individual level, it must be true that the investment szk^α is just enough to cover the population growth and depreciation of existing capital.

Golden Rule Capital Stock I

- For any steady-state per-capita capital stock k^{ss} , we know per-capita consumption c^{ss} is given by

$$c^{ss} = (1 - s)z(k^{ss})^\alpha = z(k^{ss})^\alpha - (n + d)k^{ss}$$

- Among all the steady-state per-capita capital k^{ss} , we can pick one that gives us the highest level of corresponding steady-state consumption:

$$\max_{k^{ss}} z(k^{ss})^\alpha - (n + d)k^{ss}$$

- FOC:

$$z\alpha(k^{ss})^{\alpha-1} = n + d$$

$$\implies k^{ss} = \left(\frac{n + d}{z\alpha} \right)^{\frac{1}{\alpha-1}}$$

we call this “golden-rule capital”, and from now on, denote it as k^{gr} .

Golden Rule Capital Stock II

- Note: golden-rule capital is a steady-state capital (it is the one that maximizes consumer's steady-state consumption).
- We may want to know what saving rate should the country choose if it wants to achieve maximum per-capita consumption at steady state.
- Compare the golden rule capital

$$k^{gr} = \left(\frac{n + d}{z\alpha} \right)^{\frac{1}{\alpha-1}}$$

and

$$k^{ss} = \left(\frac{n + d}{zs} \right)^{\frac{1}{\alpha-1}}$$

we know that the country can achieve its golden-rule capital by setting saving rate $s = \alpha$.

Convergence in Solow model

- Solow model is a “toy model” that allows us to study both the long-run and the short-run behavior of the economy.
- Steady state is the “long-run” trend: it gives you an idea of where the economy would end up in the far future;
- However, there is no reason to believe that a country is at steady state: there are short-run fluctuations.
- From Solow’s perspective, these short-run fluctuations are “transition dynamics”: when the economy is not at steady state, it is on its way of convergence to a long-run trend.

Convergence in Solow model: Transitory TFP Shock

- Suppose initially ($t = -1$) the economy is at steady state.
- Suppose now ($t = 0$) there is a **temporary** decrease in productivity z : z goes down only at $t = 0$ and returns to its original value at $t = 1, 2, \dots$.
- We are interested in the path of the economy after this shock:
 - Impulse: the behavior of the economy at $t = 0$ (when the shock hits);
 - Response: the behavior of the economy at succeeding periods $t = 1, 2, \dots$.

Convergence in Solow model: Transitory TFP Shock

- The key equation in our analysis is again the law of motion for per-capita capital:

$$(1 + n)k' = (1 - d)k + szk^\alpha$$

Divide $(1 + n)$ on both sides, we get an expression for tomorrow's capital:

$$k' = \frac{1 - d}{1 + n}k + \frac{sz}{1 + n}k^\alpha$$

- We can illustrate this law of motion on a (k, k') diagram.

(k, k') Diagram

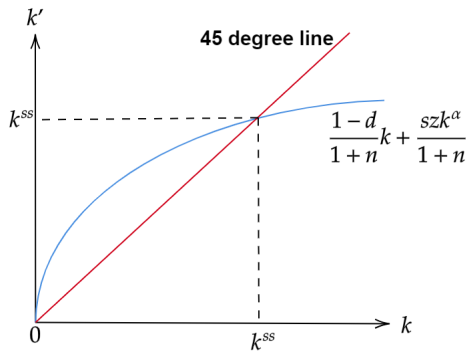


Figure: (k, k') Diagram

Convergence in Solow model: Transitory TFP Shock

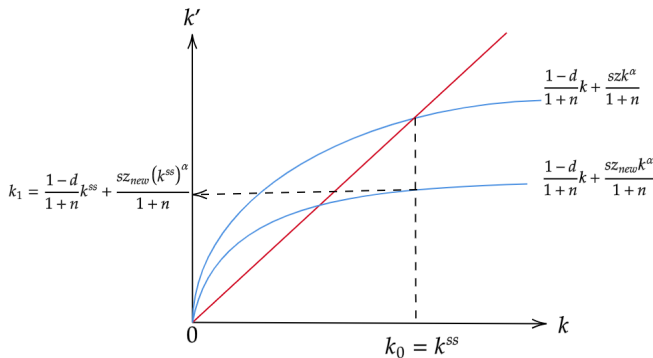


Figure: A smaller z at $t = 0$ leads k_1 to deviate from the steady state.

Convergence in Solow model: Transitory TFP Shock

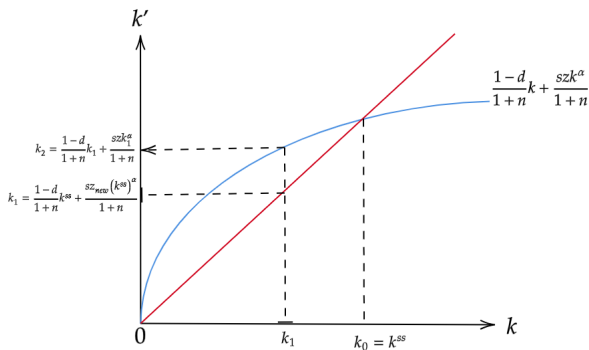


Figure: At $t = 1$, z returns to its original value, hence law of motion returns to the original one, and $k_2 > k_1$. The economy is on its path back to the original steady state.

Convergence in Solow model: Transitory TFP Shock

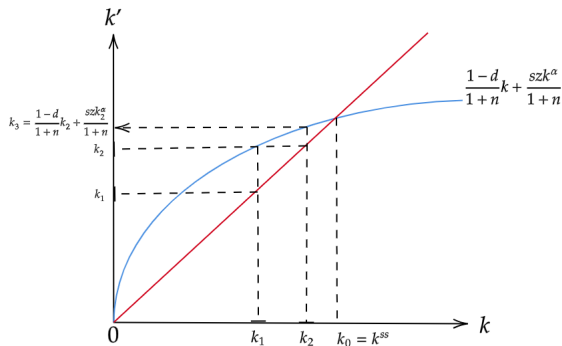


Figure: At $t = 2$, z and law of motion remain the same as the original ones, and $k_3 > k_2$. The economy is on its path back to the original steady state.

Convergence in Solow model: Transitory TFP Shock

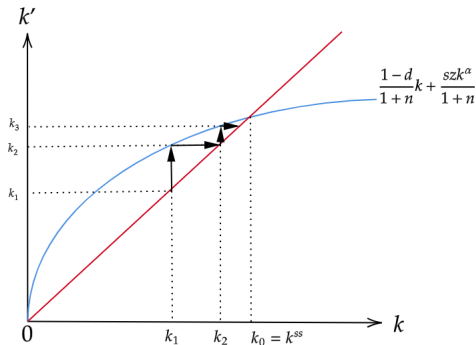


Figure: Transition dynamics on (k, k') diagram: after a transitory z decrease, the economy recovers gradually to its steady state level.

Convergence in Solow model: Transitory TFP Shock

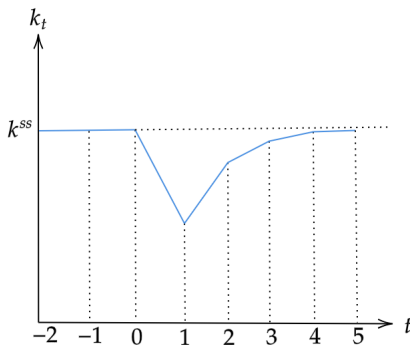


Figure: Transition dynamics on impulse-response function: after a transitory z decrease, the economy recovers gradually to its steady state level.

Convergence in Solow model: Transitory TFP Shock

Summary:

- A transitory TFP shock does **NOT** change the steady state;
- It only changes the law of motion from $t = 0$ to $t = 1$; after $t = 1$, z returns to its original value, so that the law of motion returns to the original one as well;
- Hence, starting from $t = 1$, the economy is on the path of recovering to its original steady-state level;
- As $t \rightarrow \infty$, at some point it would eventually get back to its original steady state.

Convergence in Solow model: Permanent TFP Shock

- Suppose initially ($t = -1$) the economy is at steady state.
- Suppose now ($t = 0$) there is a **permanent** decrease in productivity z : z goes down at $t = 0$ and is stuck there at $t = 1, 2, \dots$.
- Again, we are interested in the path of the economy after this shock:
 - Impulse: the behavior of the economy at $t = 0$ (when the shock hits);
 - Response: the behavior of the economy at succeeding periods $t = 1, 2, \dots$.

Convergence in Solow model: Permanent TFP Shock

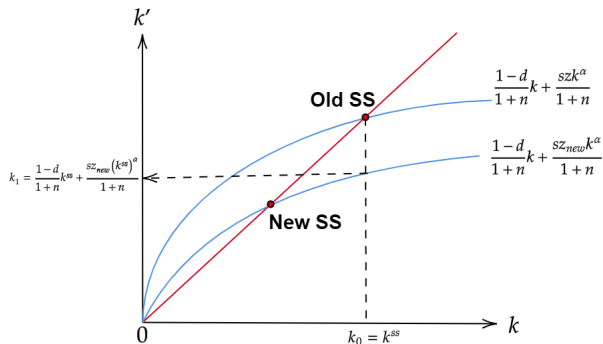


Figure: Permanent shock lowers the steady state. At $t = 0$, z decreases, and k_1 deviates from the old k^{ss} .

Convergence in Solow model: Permanent TFP Shock

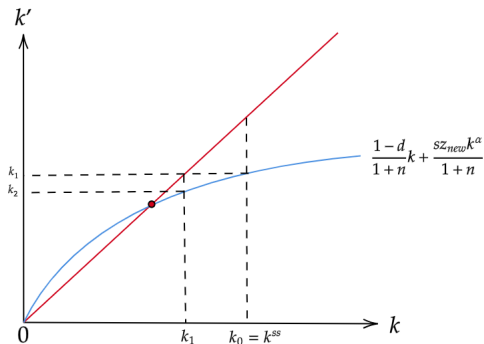


Figure: At $t = 1$, z does not return to its original value, and $k_1 > k_2$. The economy is on its path to the new steady state k_{new}^{ss} .

Convergence in Solow model: Permanent TFP Shock

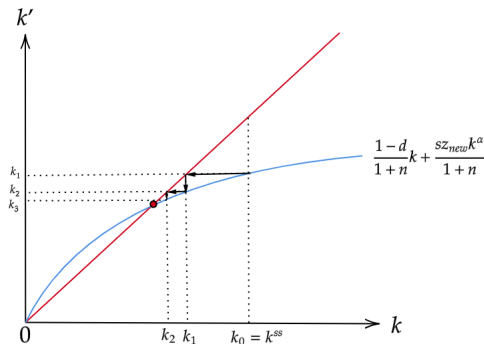


Figure: Transition dynamics on (k, k') diagram: after a permanent z decrease, the economy transitions gradually to its new steady state level k_{new}^{ss} .

Convergence in Solow model: Permanent TFP Shock

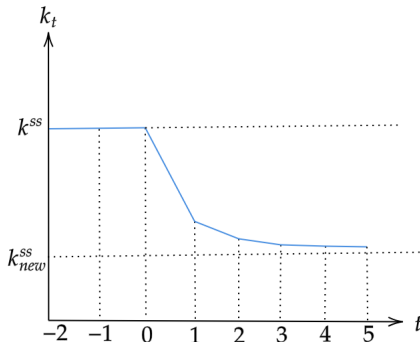


Figure: Transition dynamics on impulse-response function: after a permanent z decrease, the economy transitions gradually to its new steady state level k_{new}^{ss} .

Convergence in Solow model: Transitory TFP Shock

Summary:

- A negative, permanent TFP shock lowers the steady state;
- It changes the law of motion permanently;
- Hence, starting from $t = 0$, the economy is on the path of transitioning to its new steady-state level k_{new}^{ss} ;
- As $t \rightarrow \infty$, at some point it would eventually reach its new steady state k_{new}^{ss} .