

TA Session 8: Two-period Competitive Equilibrium

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Haoran Wang

UMD

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Roadmap

- Two-period Competitive Equilibrium;
- Exercise: news about a future productivity increase;
- Next week: review session for Midterm 2 (again, please email me in advance about the questions you want me to discuss).

Two-period Competitive Equilibrium: Consumer I

- Given prices $\{w, w', r\}$, firm's profit $\{\pi, \pi'\}$ and lump-sum tax $\{T, T'\}$, consumer chooses today's and tomorrow's consumption and leisure optimally by

$$\max_{c, c', l, l'} u(c, l) + \beta u(c', l')$$

subject to IBC:

$$c + \frac{c'}{1+r} = w(h-l) + \pi - T + \frac{w'(h-l') + \pi' - T'}{1+r}$$

- FOCs: IBC and

$$\begin{aligned} u_l(c, l)/u_c(c, l) &= w \\ u_l(c', l')/u_c(c', l') &= w' \\ u_c(c, l) &= \beta(1+r)u_c(c', l') \end{aligned}$$

Two-period Competitive Equilibrium: Consumer II

- Denote the optimal solution as $c(w, w', r)$, $c'(w, w', r)$, $l(w, w', r)$, $l'(w, w', r)$, and labor supply in both periods are given by

$$N^s(w, w', r) = h - l(w, w', r)$$

$$N^{s'}(w, w', r) = h - l'(w, w', r)$$

and the optimal saving $s(w, w', r)$ is given by

$$s(w, w', r) = w(h - l(w, w', r)) + \pi - c(w, w', r)$$

Two-period Competitive Equilibrium: Firm I

- Given prices $\{w, w', r\}$, firm chooses today's and tomorrow's labor input and tomorrow's capital by

$$\max_{N, N', K'} \pi + \frac{\pi'}{1+r}$$

where

$$\pi = zK^\alpha N^{1-\alpha} - wN - (K' - (1-d)K)$$

$$\pi' = z'(K')^{1-\alpha} (N')^{1-\alpha} - w'N' + (1-d)K'$$

- FOCs:

$$MPN = w$$

$$MPN' = w'$$

$$MPK' = r + d$$

- Denote the optimal solution as $N^d(w)$, $N^d(w')$ and $K'(r)$.

Two-period Competitive Equilibrium: Government

- Government spends on consumption goods in both periods, denoted as G and G' ;
- Government intertemporal budget constraint is given by

$$T + \frac{T'}{1+r} = G + \frac{G'}{1+r}$$

Two-period Competitive Equilibrium: Market Clearing

- In total, there are 5 markets that need to clear:
 - 3 markets today: goods, labor, bond;
 - 2 markets tomorrow: goods, labor.
- General equilibrium theory stipulates that in each period, one of the market clearing condition is redundant.
- Hence, in total we only need 3 market clearing conditions:
 - Today: goods and labor;
 - Tomorrow: labor

to solve the market clearing prices w^* , $(w')^*$, r^* .

Two-period Competitive Equilibrium: Market Clearing

- Formally, the market clearing conditions are

- Today's labor market (pins down w):

$$N^s(w, w', r) = N^d(w)$$

- Today's goods market (pins down r):

$$c + G + \underbrace{K' - (1 - d)K}_{\equiv I} \equiv Y^d = Y^s \equiv zK^\alpha (N^d)^{1-\alpha}$$

- Tomorrow's labor market (pins down w'):

$$N^{s'}(w, w', r) = N^{d'}(w')$$

Figure: Today's Labor Market

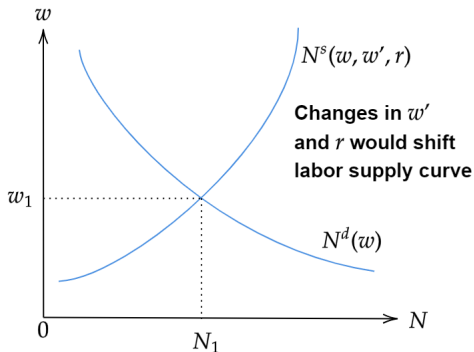


Figure: Today's labor market: pins down w^* .

Figure: Tomorrow's Labor Market

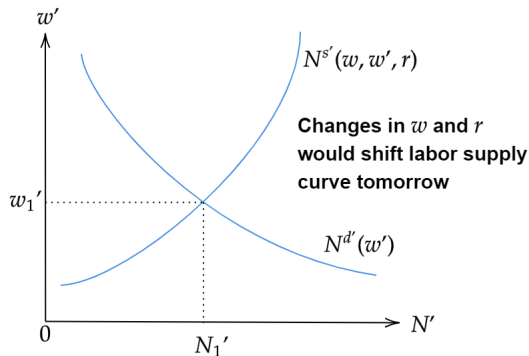


Figure: Tomorrow's labor market: pins down $(w')^*$.

Figure: Today's Goods Market

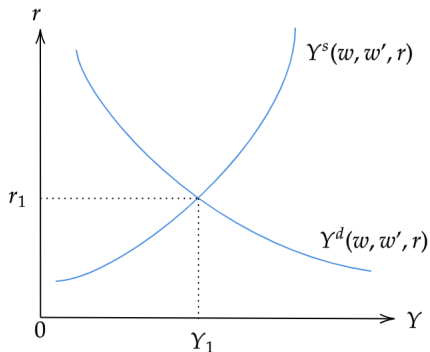


Figure: Today's goods market: pins down r^* .

Where does $Y^s = Y^d$ come from? I

- Today's goods market clearing condition comes from the consumer's today's budget constraint

$$c + s = wN^s + \pi - T$$

- Note that government debt issuance today d is

$$d = G - T$$

i.e. the deficit that the government runs today.

- Bonds market clearing condition requires that

$$s = d = G - T$$

Replace s into consumer's today's budget constraint:

$$c + G = wN^s + \pi$$

Where does $Y^s = Y^d$ come from? II

- Also, note that for the firm, today's profit π is

$$\pi = zK^\alpha (N^d)^{1-\alpha} - wN^d - I = Y^s - wN^d - I$$

Plug it into consumer's today's budget constraint:

$$c + G = wN^s + \pi = wN^s + Y^s - wN^d - I$$

- Since in equilibrium $N^s = N^d$, we have $wN^s - wN^d = 0$, hence

$$c + G = Y^s - I$$

Move I to the left, we get the goods market clearing condition.

Exercise: z' increase

- Suppose future productivity is announced to increase to $z'_2 > z'_1$.
- Follow the 4-step recipe.

Step 1: Impact

- Recall: “impact” means what would happen before anyone reoptimizes.
- z' increase would not impact any of today's variables;
- z' increases tomorrow's output, shifting $(Y^s)'$ out.

Step 2: Direct Effect

- Clearly, the direct effect of z' increase is on the firm side.
- $z'_2 > z'_1$ would
 - increase marginal product of capital: $MPK'_2 > MPK'_1 \implies I_2 > I_1$.
 - increase marginal product of labor: $MPN'_2 > MPN'_1$.
- As a result,
 - Increased investment shifts good demand curve out;
 - Labor demand curve tomorrow also shifts out,
 $(N^d)'_2 > (N^d)'_1 \implies w'_2 > w'_1$

Figure: Direct Effect on Tomorrow's Labor Market

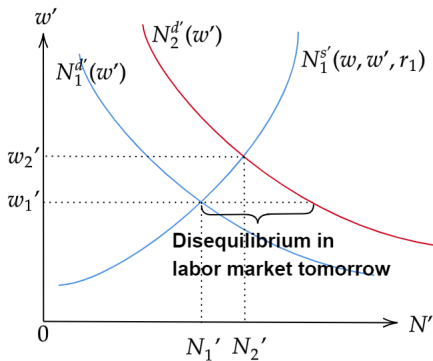


Figure: Labor Market Tomorrow, Direct Effect

Step 3a: Equilibrium Effect, first round

First, analyze how w' increase would impact today's goods and labor market:

- w' increases, making tomorrow's leisure more expensive;
- Hence, we have $l' \downarrow$, $(N^s)' \uparrow$ and $l \uparrow$, $N^s \downarrow$, $c \uparrow$.

Hence the equilibrium effects from $w' \uparrow$ are

- $N^s \downarrow$, i.e. labor supply today shifts inward.
- Meanwhile, $c \uparrow$ (from this indirect channel) + $l \uparrow$ (from direct effect) shifts good demand curve out: hence $r_2 > r_1$.
- $r \uparrow$ triggers another round of equilibrium effect!

Figure: Labor Market Today in response to $w' \uparrow$

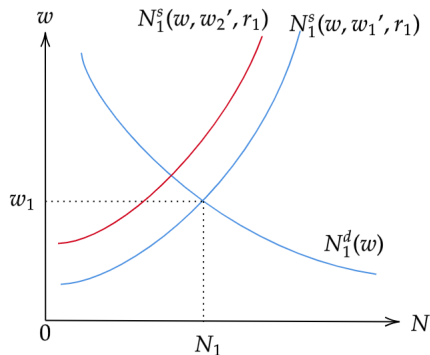


Figure: Labor Market Today, after $w' \uparrow$ and before r changes.

Figure: Goods Market Disequilibrium

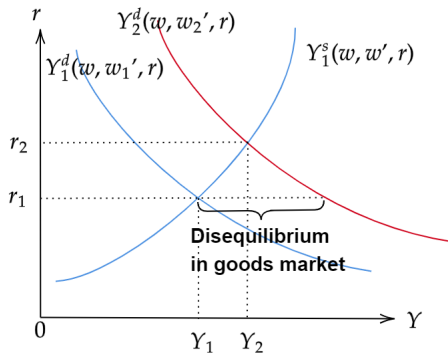


Figure: Goods Market Disequilibrium: upward pressure on interest rate r .

Step 3b: Equilibrium Effect, second round

- Increased interest rate would
 - lead the consumer to lower today's consumption c & today's leisure l , increase tomorrow's consumption c' & tomorrow's leisure l' ;
 - Hence, N^s shifts out, $(N^s)'$ shifts in;
 - Also, $r \uparrow$ lowers the marginal benefit of investment (since the firm discounts future profits more heavily) $\implies l_3 < l_2$.
 - $c \downarrow$ and $l \downarrow$ is a leftward movement along the good demand curve.
- The equilibrium effect from $r \uparrow$ on today's and tomorrow's labor market:
 - $(N^s)'$ shifts in $\implies w'_3 > w'_2, N'_3 < N'_2$ (assuming direct dominates indirect effects);
 - N^s has to shift to the right responding to $r \uparrow$.

Figure: Labor market tomorrow, after $r \uparrow$

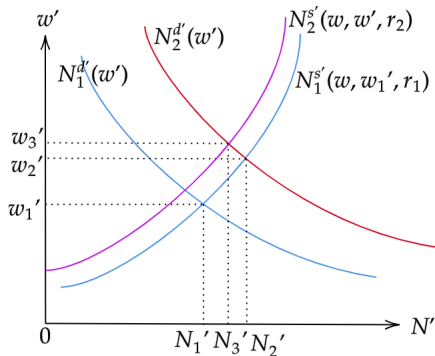


Figure: After $r \uparrow$, labor supply curve tomorrow shifts inwards.

Where would the new labor supply today end up?

- Recall that the equilibrium effect of $w' \uparrow$ shifts N^s inwards;
- Here, $r \uparrow$ shifts N^s outwards;
- Since goods market equilibrium $Y_2 > Y_1$, we must have $N_2 > N_1$ in equilibrium;
- Hence, the indirect effect from $r \uparrow$ must dominate that from $w' \uparrow$.

Figure: Labor market today, after $r \uparrow$

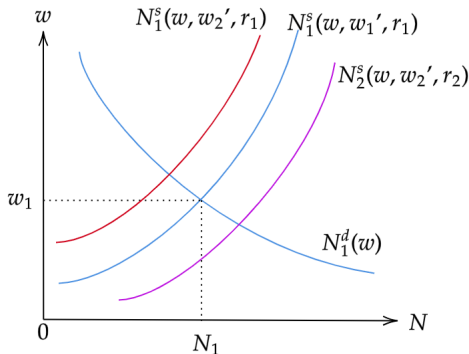


Figure: Indirect effect from $r \uparrow$ must dominate that from $w' \uparrow$ in today's labor market.

Summary

- In sum, we have
 - Today's wage w goes down, today's employment N goes up;
 - Today's output $Y^s = Y^d = Y$ goes up;
 - Today's interest rate r goes up;
 - Tomorrow's wage w' goes up, tomorrow's employment N' goes up.
(assuming direct effect dominates indirect effect)
- The direction of consumption and investment is inconclusive:
 - We know that $c + I$ must go up;
 - But it's not clear how each of c and I would change:
 - For investment I : positive direct effect (from $z' \uparrow$) and negative indirect effect (from $r \uparrow$);
 - For consumption c : positive indirect effect from $w' \uparrow$ and negative indirect effect from $r \uparrow$.