

TA Session 6: Two-period Consumer Optimization

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Roadmap

- Two-period consumer optimization problem;
- Borrowing constraint and optimal consumption;
- Marginal Propensity to Consume.

Two-period Consumption-saving Problem

- The first step is always to derive the lifetime borrowing constraint.
- With lifetime budget constraint, we can solve the optimal consumption plan with Lagrange multiplier.
- The problem becomes

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2)$$

subject to

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

Solving Optimal Consumption Plan: Steps

- 1 Set up Lagrangian:

$$\mathcal{L} = u(c_1) + \beta u(c_2) - \lambda \left(c_1 + \frac{c_2}{1+r} - y_1 - \frac{y_2}{1+r} \right)$$

- 2 First order conditions:

$$\begin{aligned} u'(c_1) &= \lambda \\ \beta u'(c_2) &= \frac{\lambda}{1+r} \\ c_1 + \frac{c_2}{1+r} &= y_1 + \frac{y_2}{1+r} \end{aligned}$$

Again, three unknowns (c_1, c_2, λ) and three equations.

Solving Optimal Consumption Plan: Steps

- ③ Combine the FOCs on c_1 and c_2 to cancel λ :

$$\frac{u'(c_1)}{\beta u'(c_2)} = 1 + r$$

Again, this is “MRS = price” condition:

- LHS is the marginal rate of substitution between tomorrow and today's consumption;
(units of tomorrow's consumption that the consumer is willing to give up in exchange for 1 unit of today's consumption.)
- RHS is the market exchange rate between tomorrow and today's consumption.

It's usually called “Euler Equation”.

- ④ Combine the Euler Equation with the budget constraint, solving the system of 2 equations with 2 unknowns (c_1, c_2), we get c_1^* and c_2^* .

Graphical Illustration of Optimal Consumption Plan

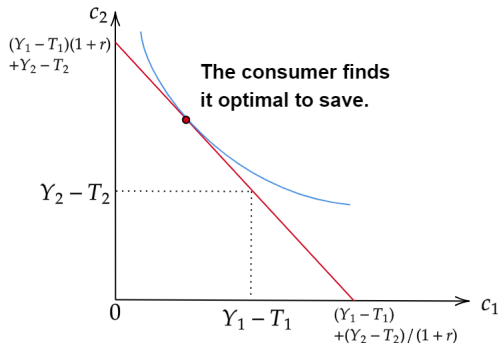


Figure: Case 1: the consumer find it optimal to save.

Graphical Illustration of Optimal Consumption Plan

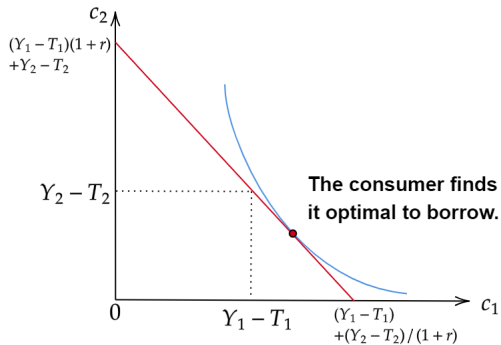


Figure: Case 2: the consumer find it optimal to borrow.

An Example: $\beta(1+r) = 1$

- We are interested in a special case $\beta(1+r) = 1$;
(i.e. subjective discount factor coincides with market discount rate.)
- When $\beta(1+r) = 1$, Euler equation becomes

$$\frac{u'(c_1)}{\beta u'(c_2)} = (1+r) \implies u'(c_1) = u'(c_2)$$

and hence $c_1 = c_2$.

- In other words, the consumer **perfectly smooth** her consumption over time.
- Plug it into budget constraint, solve the optimal consumption plan (c_1^*, c_2^*) :

$$c_1^* = c_2^* = \frac{1+r}{2+r} \left(y_1 + \frac{y_2}{1+r} \right) = \frac{1/\beta}{1/\beta + 1} (y_1 + \beta y_2)$$

An Example: $\beta(1+r) = 1$

$$c_1^* = c_2^* = \frac{1/\beta}{1/\beta + 1} (y_1 + \beta y_2) = \frac{1}{1 + \beta} y_1 + \frac{\beta}{1 + \beta} y_2$$

- If $y_1 < y_2$, then

$$c_1^* = \frac{1}{1 + \beta} y_1 + \frac{\beta}{1 + \beta} y_2 > y_1, c_2^* < y_2$$

and hence the consumer is a borrower.

- If $y_1 > y_2$, then

$$c_1^* < y_1, c_2^* > y_2$$

and hence the consumer is a saver.

Borrowing Constraint

- The key assumption behind intertemporal budget constraint: the consumer can save or borrow by any amount she wishes.
- In other words, s can be any positive or negative real numbers.
- Now, suppose we deviate from the assumption: the consumer is not allowed to borrow, i.e. $s \geq 0$.
- How would this change the intertemporal budget constraint?
- We still have

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

and an additional constraint

$$c_1 \leq y_1$$

since $s = y_1 - c_1 \geq 0$.

Graphical Illustration

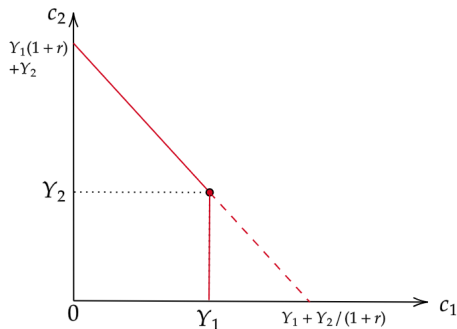


Figure: Intertemporal budget constraint + borrowing constraint

Graphical Illustration

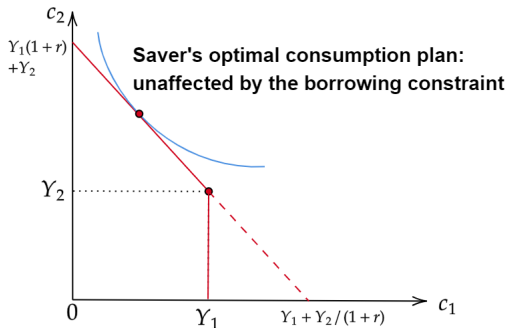


Figure: Saver's optimal plan is not affected by the borrowing constraint

Graphical Illustration

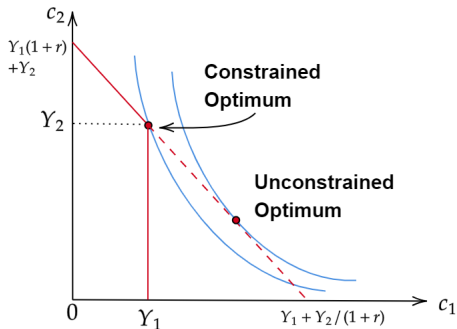


Figure: Borrower's optimal plan is to consume the endowment in each period.

Borrowing Constraint: Summary

- People borrow if they
 - ① believe that their future income would be higher, enough to repay the debt;
 - ② want to smooth consumption between now and future.
- With borrowing constraint, this consumption-smoothing motive cannot be satisfied.
 - Borrowing is not allowed in our example;
 - This impedes consumers who have low income today and high income tomorrow from borrowing.
- As a result, it's optimal for the borrowers to consume their endowments under borrowing constraint.
- In this case, the Euler equation is violated:

$$u'(c_1) > \beta(1+r)u'(c_2)$$

Marginal utility of consumption today is greater than marginal utility of consumption tomorrow, so the consumer wants to increase c_1 by borrowing but he can't.

Marginal Propensity to Consume

- Economists are interested in how consumers allocate their income to consumption and saving.
- Suppose there is a change in current income, denoted as Δy_1 .
- Optimal consumption plan would change accordingly.
- Marginal Propensity to Consume (MPC) is defined as

$$MPC_1 \equiv \frac{\Delta c_1}{\Delta y_1}$$

$$MPC_2 \equiv \frac{\Delta c_2}{\Delta y_1}$$

- It captures the fraction that the consumer would spend additionally on consumption for each unit of additional current income.

MPC: Transitory vs Permanent

- Suppose $\beta(1+r) = 1$, we have

$$c_1^* = c_2^* = \frac{1}{1+\beta}y_1 + \frac{\beta}{1+\beta}y_2$$

- Suppose the income shock is **transitory**, i.e. only y_1 goes up by Δy :

$$c_1^{new} = c_2^{new} = \frac{1}{1+\beta}(y_1 + \Delta y) + \frac{\beta}{1+\beta}y_2$$

hence

$$MPC_1 = \frac{c_1^{new} - c_1^*}{\Delta y} = \frac{\frac{1}{1+\beta}\Delta y}{\Delta y} = \frac{1}{1+\beta}$$

and

$$MPC_2 = \frac{c_2^{new} - c_2^*}{\Delta y} = \frac{1}{1+\beta}$$

MPC: Transitory vs Permanent

- Suppose $\beta(1+r) = 1$, we have

$$c_1^* = c_2^* = \frac{1}{1+\beta}y_1 + \frac{\beta}{1+\beta}y_2$$

- Suppose the income shock is **permanent**, i.e. both y_1 and y_2 goes up by the same amount Δy :

$$c_1^{new} = c_2^{new} = \frac{1}{1+\beta}(y_1 + \Delta y) + \frac{\beta}{1+\beta}(y_2 + \Delta y) = c_1^* + \Delta y$$

hence

$$MPC_1 = \frac{c_1^{new} - c_1^*}{\Delta y} = \frac{\Delta y}{\Delta y} = 1$$

and

$$MPC_2 = \frac{c_2^{new} - c_2^*}{\Delta y} = 1$$

MPC: Transitory vs Permanent

- This simple example illustrates how consumer would respond differently to a transitory and a permanent income shock:
 - MPC of a **transitory** income shock is typically **smaller** than MPC of a **permanent** income shock;
 - In a **transitory** income shock: the consumer wants to smooth consumption $\implies$ “spread out” the income increase in both periods;
 - In a **permanent** income shock: the consumer knows that his future income will increase as well, and his consumption today depends on both today and tomorrow’s income
 $\implies$ spend a higher fraction of Δy than in transitory shocks.
- This clarifies page 22 of lecture 9.

MPC: Constrained vs Unconstrained

- Again, suppose $\beta(1+r) = 1$, so that

$$c_1^* = c_2^* = \frac{1}{1+\beta}y_1 + \frac{\beta}{1+\beta}y_2$$

- Suppose there is a **transitory** increase in y_1 , denoted as Δy ;
- If there is no borrowing constraint:
 - Consumption smoothing $\implies \text{MPC} = \frac{1}{1+\beta} < 1$;
- If there is borrowing constraint:
 - If the consumer is a saver and not affected by the borrowing constraint, $\text{MPC} < 1$;
 - If the consumer is a borrower and has to consume the endowment ($c_1^* = y_1$, $c_2^* = y_2$) under the borrowing constraint,

$$\text{MPC}_1 = \frac{c_1^{\text{new}} - c_1^*}{\Delta y} = \frac{y_1 + \Delta y - y_1}{\Delta y} = 1$$

MPC: Constrained vs Unconstrained

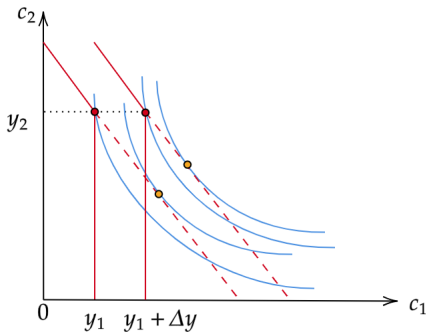


Figure: $MPC = 1$ for a borrowing-constrained consumer

- Measuring MPC in data: investigate Δc in response to an exogenous, transitory shock in income (e.g. lottery gains, tax rebate, etc.)
- US Aggregate time series of consumption and income: c_t is about a third as volatile as y_t , indicating consumption smoothing.
- Micro-level data: the quarterly MPC (change in consumption in a quarter) out of a 500-dollar windfall is typically between 0.15 and 0.25;
($\implies$ consistent with consumption smoothing?)
- Micro-level data: MPC varies a lot across individuals, with some MPCs close to 0 and some close to 1;
 - Consumers subject to credit/borrowing constraints usually have less holding of cash/liquid assets;
 - these consumers present a higher MPC than unconstrained consumers.