

TA Session 7: Two-period Firm Optimization

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- Review of Firm's Optimization Problem;
- Firm's optimization problem with imperfect conversion of investment;
- Firm's optimization problem with $p_k \neq 1$.

Firm's Optimization Problem

- Firm takes the following variables as given:
 - today's capital K ,
 - today's productivity z and tomorrow's productivity z' ,
 - today's wage rate w and tomorrow's wage rate w' ,
 - interest rate r ,
 - depreciation rate d .
- Firm chooses today's labor input N , tomorrow's labor input N' and tomorrow's capital K' to

$$\max \left\{ zK^\alpha N^{1-\alpha} - wN - I + \frac{z'(K')^\alpha (N')^{1-\alpha} - w'N' + (1-d)K'}{1+r} \right\}$$

where investment I is

$$I = K' - (1-d)K$$

Firm's Optimization Problem

$$\max_{N, N', K'} \left\{ zK^{\alpha} N^{1-\alpha} - wN - (K' - (1-d)K) + \frac{z'(K')^{\alpha} (N')^{1-\alpha} - w'N' + (1-d)K'}{1+r} \right\}$$

- FOCs:

$$\begin{aligned}(1-\alpha)zK^{\alpha} N^{-\alpha} &= w \\ (1-\alpha)z'(K')^{\alpha} (N')^{-\alpha} &= w' \\ -1 + \frac{\alpha z'(K')^{\alpha-1} (N')^{1-\alpha} + 1-d}{1+r} &= 0\end{aligned}$$

- The last equation can be written as

$$\alpha z'(K')^{\alpha-1} (N')^{1-\alpha} = r + d$$

Firm's Optimization Problem

- Again, marginal benefit = marginal cost.
 - FOCs on labor: marginal product of labor (MPN) = marginal cost of labor (wage);
 - FOC on tomorrow's capital:
 - marginal cost: investing leads today's profit to drop;
 - marginal benefit: investing $\implies$ tomorrow's marginal product of capital $\uparrow$, resale value of tomorrow's capital $\uparrow$.
 - At the optimum, marginal benefit = marginal cost.

Exercise 1: Imperfect Conversion of Investment

- Suppose that we modify the law of motion of capital such that only a fraction of our investment is converted the capital stock tomorrow:

$$K' = (1 - d)\bar{K} + (1 - \Phi)I$$

- Consider the metaphor: an iceberg melts when transported, so only a fraction of the starting amount arrives at the destination.
- Here Φ is an iceberg trade cost where one unit of investment increases by the capital stock for tomorrow by a fraction $1 - \Phi$ of a unit.

Profit maximization problem

- The firm choose N , N' , and K' to maximize the present value of profits:

$$\max_{N, N', K' \geq 0} F(\bar{K}, N) - wN - I + \frac{F(K', N') - w'N' + (1 - d)K'}{1 + r}$$

- Substituting out investment using our law of motion of capital:

$$\max_{N, N', K' \geq 0} F(\bar{K}, N) - wN - \frac{K' - (1 - d)\bar{K}}{1 - \Phi} + \frac{F(K', N') - w'N' + (1 - d)K'}{1 + r}$$

First order condition and comparing investment rules

- Taking the derivative of the profit maximization problem with respect to K' :

$$-\frac{1}{1-\Phi} + \frac{MPK' + (1-d)}{1+r} = 0 \implies \frac{MPK' + (1-d)}{1+r} = \frac{1}{1-\Phi}$$

Denote the optimal tomorrow's capital as $(K')^*$.

- Note that our benchmark case:

$$\frac{MPK' + (1-d)}{1+r} = 1$$

Denote the optimal tomorrow's capital as $\bar{K}'$.

- How does $\bar{K}'$ compare with $(K')^*$?

Graphical Illustration

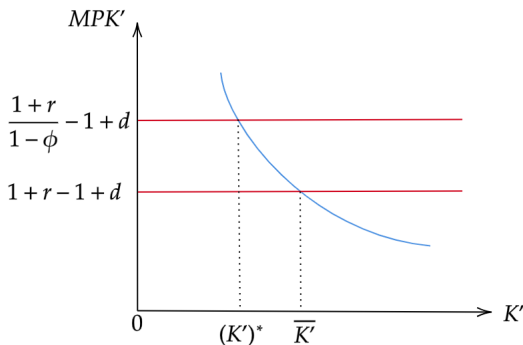


Figure: Imperfect conversion discourages investment: $\bar{K}' > (K')^*$.

- Back to the FOC:

$$\frac{MPK' + (1 - d)}{1 + r} = \frac{1}{1 - \phi}$$

- We can see that $\phi > 0$ makes the marginal cost of investment higher.
- If the investment is at the benchmark level $\bar{K}'$, then marginal benefit of investment would be smaller than the marginal cost of investment;
- As a result, it's tempting for the firm to scale back investment.

Example 2: $p_k \neq 1$

- Suppose any capital the firm has remaining at the end of the period can be sold at the price p_k ;
- Here, 1 is the price of consumption goods and p_k is the relative price of capital to consumption goods;
- This modification will appear in the firm's profit π' :

$$\pi' = F(K', N') - wN' + p_k(1 - \delta)K'$$

and hence the present value of a firm's profits $V = \pi + \frac{\pi'}{1+r}$

Profit maximization problem and first order conditions

- The firm choose N , N' , and K' to maximize the present value of profits:

$$\max_{N, N', K' \geq 0} F(\bar{K}, N) - wN - K' + (1 - d)\bar{K} + \frac{F(K', N') - w'N' + p_k(1 - d)K'}{1 + r}$$

- The first order condition with respect to K' is:

$$-1 + \frac{MPK' + p_k(1 - d)}{1 + r} = 0 \implies \frac{MPK' + p_k(1 - d)}{1 + r} = 1$$

Denote the optimal tomorrow's capital $(K')^{**}$.

- How does $(K')^{**}$ compare to the benchmark tomorrow's capital $\bar{K}'$?

Graphical Illustration

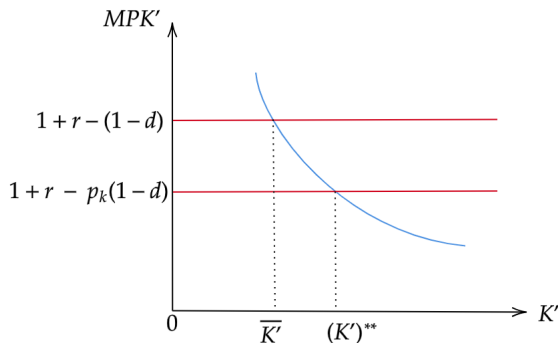


Figure: If $p_k > 1$, the firm would invest more: $\bar{K}' < (K')^{**}$.

Comparing investment rules

- If $p_k > 1$, the firm would invest more than the benchmark case:
 $(K')^{**} > \bar{K}'$.
- If $p_k < 1$, the firm would invest less than the benchmark case:
 $(K')^{**} < \bar{K}'$.
- Intuition:
 - p_k changes the marginal benefit of investment;
 - If $p_k > 1$, the resale value of capital is high, which gives the firm extra incentive to increase capital stock tomorrow (and sell it after production);
 - If $p_k < 1$, the resale value of capital is low, which lowers the incentive to increase capital stock tomorrow (and sell it after production).